

Transport Phenomena

The Art of Balancing

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Preface

In 1956, Hans Kramers in Delft published his first lecture notes (in Dutch) on ‘Fysische Transportverschijnselen’ – to the best of our knowledge the first systematic treatment of the emerging discipline of Transport Phenomena. In 1958, Robert B. Bird spent a period in Delft as a guest of Hans Kramers. This visit gave the two Professors the excellent opportunity to explore and improve the way of teaching Transport Phenomena. Bird published his ‘Notes on Transport Phenomena’ in the Fall of 1958, followed in 1960 by the first Wiley edition of the famous ‘Transport Phenomena’ textbook by Bird, Stewart & Lightfoot.

In Delft, the Dutch students kept using Kramers’ shorter lecture notes in Dutch which in the course of the years were continuously improved, also by Kramers’ successors Wiero J. Beek and John M. Smith. All those years the analogy of momentum, heat and mass transport remained the leading theme, just like in Bird’s textbook. An essential element in the way Transport Phenomena has been taught in Delft has always been the emphasis on developing the students’ ability of solving realistic engineering problems. Over the years, hundreds of challenging exam problems were devised.

In 1996, the current authors published a new version of the Delft lecture notes on Transport Phenomena for various reasons. Students, their interests, their prior education, and the way they prepare for exams were changing. In many curricula, the course got a different role and place. New applications in biotechnology, biomedical, smart materials and solar developed – remote from the traditional chemical industry. And Computational Fluid Dynamics (CFD) developed into a real analytical tool. All this required different didactic methods for teaching as well as different examples and exam problems.

Our new version of the Delft textbook ‘Transport Phenomena’ (still in Dutch) built on the earlier Delft lecture notes but was still based on the classical analogy of momentum, heat and mass transport, although the order of treatment was changed: fluid mechanics largely moved to the end, provoked by ideas developed by Kees Rietema at Eindhoven University of Technology. Most importantly, however, we put a much stronger emphasis on the basic method of drawing up balances, either about a particular device (a macro-balance) or about a differential element anywhere in a material or fluid (a micro-balance). In most cases, such a balance turns into a differential equation. We believe that teaching students as to how to draw up balances and solve differential equations is an excellent preparation for exploiting

modern CFD techniques. The exam requirement that students should be capable of solving original problems was maintained.

Our textbook was quite successful: a 2nd edition was released in 2003-2005, a 3rd edition in 2008. In recent years, however, increasing numbers of foreign students arrived at Delft University of Technology for various MSc programs. This development has prompted the idea of publishing an English version of our Dutch textbook, simultaneously updating and improving a Dutch 4th edition. This textbook – Transport Phenomena: The Art of Balancing – is the result. We hope it will find its way to foreign universities as well.

We like to express our sincere appreciation for all suggestions for improvements we received over the years. In particular we like to acknowledge the contributions from A.G.N. Boers, C. Ouwerkerk, G.C.J. Bart, C.R. Kleijn, J.J.Ph. Elich, L.M. Portela, J.A. Battjes, R.B. Bird, and J.E. Schievink. We remain open for suggestions of further improvements.

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1 Balances

1.1 The balance: recipe and form

The field of transport phenomena covers the transport of the three most important quantities – mass, energy, and momentum – in any (physical or chemical) process. The addition of the words ‘in any process’ in particular is an indication of one of the most important features of the field: transport phenomena is, above all, an engineering field with a wide range of applications.

Nonetheless, the field is also fundamental, given that it forms the basis for many other chemical engineering disciplines, such as reactor engineering, separation technology, and fluid mechanics. This makes transport phenomena a must for any chemical engineer. A good knowledge of the subject is also very useful to those in other professions, such as mechanical, mining, civil, and building engineers, physicists, chemists, and materials scientists.

The area covered by the field of transport phenomena and the discipline of chemical engineering is considerable. There are, for example, all kinds of processes in the chemical and petro-chemical industry, the flow of one or more phases through a pipeline, the behaviour of bubbles in a bioreactor, or the filling of a casting mould with liquid metal. At the other end of the scale, the field is also very important to more everyday matters, such as the heat emission of a radiator and the associated air flows in the room, and transport of oxygen by blood flow. Fortunately, these very different processes can be clearly understood and described with a limited number of rules.

Flow phenomena and heat and mass transfer are described in this field in terms of continuum properties, with only occasional references to molecular processes. This is how the basis is laid for chemical engineering: the expertise of designing and improving processes in which substances are transported, transformed, processed, or shaped. It is important here to fully understand the essence of a process – that is, to identify the essential stages in the transport of mass, heat, and/or momentum. The transport of these three quantities can, as it happens, be described in exactly the same way. Transport phenomena lays the basis for physical technology and provides the necessary tools. This textbook is about these tools.

First and foremost, transport phenomena is a subject of *balances* and *concepts* by which physical processes and phenomena can be described. In many cases, the subject is about deviations from a state of equilibrium and the subsequently occurring *resistances* to heat and mass transport. It frequently concerns a quantitative description of cause and effect. With the help of these still somewhat vague terms, it is possible to gain an outlined, but also very detailed, understanding and description of the aforementioned and countless other processes. This chapter will discuss the term *balance* in extensive detail.

For the description of the transport of any quantity, such as the transport of oxygen from bubbles to the liquid phase in a fermenter or the transport of heat through the wall of a furnace, the balance is an essential tool. The basic principle of the balance is the bookkeeping of a selected physical quantity. This concept is of particular importance when working with what are known as conserved quantities; these are quantities (like mass and energy) that are not lost during a process, but conserved.

The field of transport phenomena deals with steady-state or transient (time-dependent) processes in which mass, energy and momentum are exchanged between domains as a result of driving forces (differences in concentrations of mass, energy and momentum, and/or in pressure). Transport phenomena is therefore primarily about the ‘bookkeeping’ of the three physical quantities: mass, energy and momentum.

This bookkeeping can refer to large control volumes, which involve *macrobalances*; however, balances can also be drawn up in relation to very small control volumes – these are known as *microbalances*, which provide information at a local scale. In almost all cases, solving problems such as about transport or transfer rates, or about changes in concentrations or temperatures, starts with drawing up one or more balances.

The next step is to derive from such balances proper equations, in many cases differential equations; the latter require initial and/or boundary conditions. The final step is about solving these (differential) equations to find the answer to the problem under consideration. In this approach, it is essential to denote all quantities with symbols!

The *general recipe* for drawing up a balance and solving the problem can be summarised as follows:

- 1) Make a sketch of the situation. Use symbols rather than numerical values to indicate quantities.
- 2) Select the quantity G that is being transported or transferred in the process under consideration.
- 3) Select the ‘control volume’ V about which information is to be obtained.

- 4) Find out whether and if so, how, the quantity of G in the control volume V changes during a brief period of time Δt . Draw up the balance (using symbols).
- 5) Solve the (differential) equation resulting from the balance.

The quantity of G in V can change in all kinds of ways. These should be examined systematically and, if applicable, included in the balance. For example, during Δt , G can flow into V from outside. As a result, the quantity of G inside V increases. It is also possible for G to flow outwards, from inside V . In this case, the quantity of G in V falls. We refer to “inflow” and “outflow”, respectively. Of course, it is also possible for *production* of G to occur inside V during period Δt : as a result, the total quantity of G in V increases. Negative production (= destruction, consumption, annihilation) is also possible, for example if G stands for the mass of a reagent that is being transformed in a chemical process.

Bear in mind that G may not necessarily be the quantity in which you are interested. In order to calculate temperature T , for example, a thermal energy balance has to be drawn up, and the thermal energy U must be selected for G .

The general structure for a balance is now as follows (see also Figure 1.1):

$$\begin{aligned} & \text{The change of } G \text{ in } V \text{ during } \Delta t = \\ & = G \text{ (at time } t + \Delta t \text{) in } V - G \text{ (at time } t \text{) in } V \\ & = \text{ quantity of } G \text{ that flows from outside into } V \text{ during } \Delta t + \\ & \quad - \text{ quantity of } G \text{ that flows outside from inside } V \text{ during } \Delta t + \\ & \quad + \text{ net quantity of } G \text{ that is produced in } V \text{ during } \Delta t \end{aligned}$$

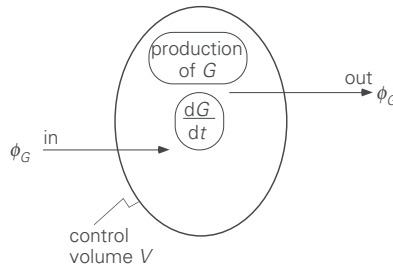


Figure 1.1.

From now on, the symbol ϕ will be used to denote a transport (rate), with the dimension ‘quantity of G per unit of time’. Instead of transport rate, the term *flow rate* is used. The letter P stands for net production per unit of time, or net production rate. With the help of this notation, the quantity of G that flows ‘inwards’ (= from the outside to the inside) *during* the period of time Δt can, if Δt is very short, be written as the product of the flow rate ‘in’ *at time* t and the period of time Δt :

2

Mechanisms, non-dimensional numbers, forces

2.1 Molecular transport

2.1.1 *Moving molecules*

So far, we have looked primarily at flows of mass, energy, and momentum resulting from ‘collective’ behaviour. Mass of a certain species, energy of any type, or momentum flowed into (or out of) the control volume because it was carried, as it were, ‘under the arm’ of a mass flow rate that flowed into (or out of) the control volume. It could be said that this involved the molecules going ‘together’. The form of the expressions for the inflows and outflows is always a product of a mass (or volume) flow rate and a concentration in that flow rate (per unit of mass or volume). Such flows are forms of *convective transport*.

However, individual molecules are also able to effect transport of mass, energy, and momentum. At a molecular scale, molecules in a liquid or gas move around somewhat chaotically, criss-crossing and colliding with each other as a result of their thermal motion. Each molecule transports its own mass, momentum, and kinetic energy. Because they collide, they are able to transfer their momentum and energy wholly or partly to other molecules. In time, net transport over larger distances is possible. The mobility of individual molecules in a solid is, of course, much less, but here too, due to the fact they vibrate they are perfectly able to pass, or conduct, heat by colliding with neighbouring molecules.

Notwithstanding the fact that this *molecular transport* is caused by the movements of individual molecules, it is useful to talk in terms of ‘mean quantities’ (continuum quantities) like density, (mass) concentration, and temperature. This Section describes the aforementioned transport in a phenomenological manner.

Diffusion

The following experiment serves to give us a greater understanding of the mechanism of molecular transport. A container is divided into two equal parts by a partition, in which there is a small hole (see Figure 2.1). The right-hand side of the tank is filled completely with distilled water, and the left-hand side with saltwater. The pressures on both sides are equal, which means no convective flow occurs from one compartment to the other. However, after a long enough period of time, both

sides will contain equal levels of saltwater. This equilibrium situation will never return to the first situation in which one side contained distilled and the other saltwater. What is the explanation for this? A close-up of the hole (see Figure 2.2) makes this clear.

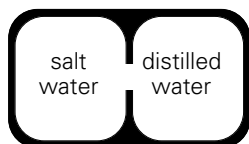


Figure 2.1.

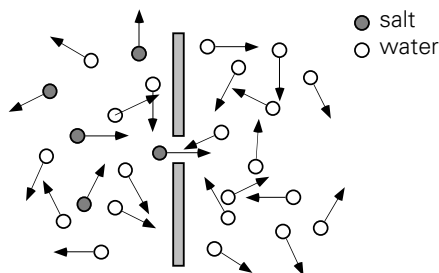


Figure 2.2.

A veritable bombardment of molecules takes place from both sides onto the partition wall, and thus also onto the imaginary plane of the hole: as a result, as many molecules find their way through the hole from left to right as they do from right to left, maintaining the zero pressure difference between the two compartments. However, if we look at the salt molecules⁴, things are different. The number of salt molecules approaching the hole from either side depends on the number density (concentration) of salt molecules on that side. At first, salt molecules only travel through the hole from left to right – there are no salt molecules on the right, after all. In time, the number of salt molecules in the right compartment increases, and so does the number of salt molecules that is able to return to the left. The *net* transport from left to right gradually decreases until the concentrations of salt are equal on both sides.

The process described above is known as *molecular diffusion* and originates in macroscopic differences in concentration. It is to be expected that the resulting net mass transport rate ϕ_m simply depends on this difference in concentration: the greater the difference, the stronger the net transport rate, and the greater the distance between the places of high and low concentration, the weaker the transport rate (as it may take the molecules longer to cover a larger distance).

Conduction

Molecular heat transport goes by analogy. The driving ‘force’ here is a difference in temperature. Temperature is actually the resultant of the kinetic energy of a very

⁴ It is actually incorrect to talk of salt molecules, as the dissolution of salt always involves dissociation into ions. This is a typical example of the use of an ‘engineering point of view’: the ‘removal’ of the physical reality and the introduction of all kinds of simplifications to the phenomenon under review in order to highlight and describe the essence of the phenomenon under consideration.