Electrical Networks

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Preface

The book begins with simple basic concepts and the principal circuit theorems, which form a good link to the knowledge of the starting student; initially the mathematics is not difficult either. Very soon the controlled sources (including opamps) are introduced.
Subsequently the whole theory is extended to alternating currents including complex voltages, currents and impedances, while the mathematics becomes increasingly complicated. The subsequent chapters discuss transformers, three-phase systems, Fourier analysis, the complex frequency, poles and zeros, two ports including filters and networks with switches (transient response). Then an extensive chapter on computer aided design follows; it turns out that the problems dealt with before can be solved by computer.
Each chapter finishes with a large number of problems with increasing difficulty. The book concludes with the answers to the problems.
In the field of network theory one cannot avoid a great influence of mathematics, but in many places I explain the physical background of mathematical results.
In order to avoid calculations that are too complex I have in most cases used simple values for the network elements.

I wish to express my appreciation to Professor K.M. Adams and Professor P. Dewilde for the many informative conversations I had with them and to my colleague W. Buijze for his critical remarks and because he was so kind as to offer me a number of instructive problems.
Finally I wish to thank J. Schievink (VSSD) for his splendid and friendly cooperation and his suggestions.

Delft, December 1989 A. Henderson

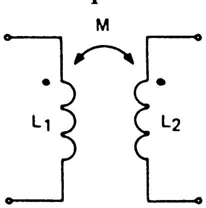
Contents

Symbols

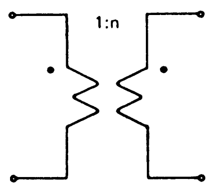
		unit
Admittance matrix	$\mathcal{Y}$	S
Admittance	Y	S
Ampere turns	(AT)	A
Amplitude and modulus of the boltage	|V|	V
Amplitude and modulus of the current	|I|	A
Angular frequency	ω	rad/s
Apparent power	|S|	VA
Argument of the impedance Z	arg Z	rad
Average, real power	P	W
Bandwidth	B	Hz or rad/s
Capacity	C	F
Cascade matrix	$\mathcal{K}$	
Coefficient of coupling	k	
Complex frequency	λ	s^{-1}
Complex power	S	VA
Conductance	G	S
Conductancematrix	$\mathcal{G}$	S
Conjugate complex of Z	Z*	Ω
Constant charge	Q	C
Constant, or complex current	I	A
Constant, or complex voltage	V	V
Coupled flux	Φ	Vs
Dampingexponent	σ	s^{-1}
Determinant of the matrix	det $\mathcal{Z}$ or $\|\mathcal{Z}\|$	
Detuning	d	
Differentiating operator	p	s^{-1}
Effective value of the current I	I_{eff}	A
Electric field	E	V/m
Energy	W	J
Force	F	N
Frequency	f	Hz
Hybrid matrix	$\mathcal{H}$	
Impedance matrix	$\mathcal{Z}$	Ω
Impedance	Z	Ω
Inductor	L	H
Integrating operator	p^{-1}	s
Leak	σ	

Magnetic field	H	A/m
Magnetic fluxdensity	B	Vs/m^2
Magnetic resistance	R_m	A/Vs
Momental charge	q	C
Momental current	i	A
Momental power	p	W
Momental voltage	v	V
Mutual induction	M	H
Period	T	s
Permeability of the vacuum	μ_0	Vs/Am
Phase	φ or p	rad
Quality of an inductor	Q	
Reactance	X	Ω
Reactive power	Q	VA_r
Relative permeability	μ_r	
Resistance matrix	$\mathcal{R}$	Ω
Resistance	R	Ω
Reverse cascade matrix	$\mathcal{J}$	
Reverse hybrid matrix	$\mathcal{G}$	
Specfic resistance	ρ	$\Omega/m/mm^2$
Specific conductance	γ	$S/m/mm^2$
Susceptance	B	S
Time constant	τ	s
Voltage or current ratio	H	
Voltage ratio in decibel	G	dB
Voltage	$V_{ab} = V_a - V_b$	V

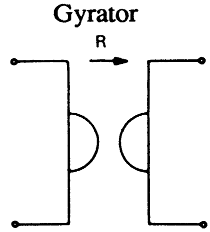
Magnetic coupled inductors



Transformer

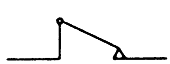


Gyrator

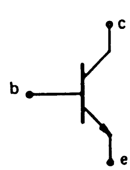


Switch:
make contact

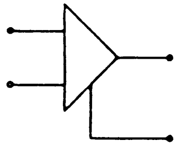
Switch:
break contact



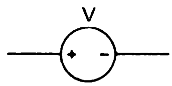
Transistor



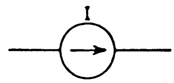
Operational amplifier



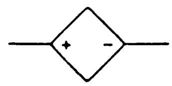
Voltage source



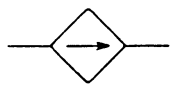
Current source



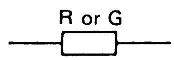
Controlled voltage source



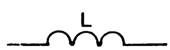
Controlled current source



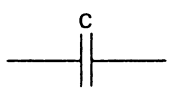
Resistor or conductor



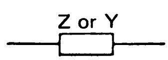
Inductor



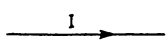
Capacitor



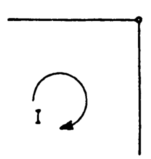
Impedance or admittance



Branch current

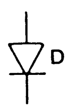


Mesh current



Non-lineair resistor

Diode



1

d.c. currents and d.c. voltages

1.1 Current, potential, voltage and resistance

The unit of electric *current* is the *ampere*. If a *charge* of 1 *coulomb* (1 C) is passed through the cross-section of a conductor in 1 *second* (1 s), the current will be 1 *ampère* (1 A). The current is indicated by I or i, so if Q is charge and T is time we have

$$I = \frac{Q}{T}. \tag{1.1}$$

We will use capital letters if the quantity is constant and lower case letters if the quantity varies with time. In the latter case for the current we get:

$$i = \frac{dq}{dt}. \tag{1.2}$$

The *potential* V_A (measured in *volt*, V) at a point A is equal in magnitude to the *energy* (measured in *joule*, J) required to take a charge of 1 C from a point where the potential is regarded as zero (V = 0) to point A.
The *voltage* V_{AB} between two points A and B is the energy that has to be supplied to take a charge of 1 C from point B to point A. So we have:

$$V_A = V_B + V_{AB},$$

thus

$$V_{AB} = V_A - V_B. \tag{1.3}$$

It follows that

$$V_{BA} = -V_{AB}. \tag{1.4}$$

We assign to each wire in a network a direction and we call this the *positive current direction*. We use this as a reference. One can freely choose this positive current direction. Once chosen, the sign of the current is known.
In Figure 1.1 a wire is drawn with the positive current direction I_1. It is possible that there is no current in the wire, but we still maintain the arrow with the indication I_1, I_1 being zero. If there is a current in the wire of 7 A from left to right we write $I_1 = 7$ A; if there is a current in the wire of 5 A from right to left we write $I_1 = -5$ A.

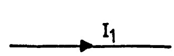


Figure 1.1

In Figure 1.2 a situation is represented in which the current direction changes. Between 0 and t_1 seconds the current i_3 is positive, between t_1 and t_2 the current i_3 is negative.
For the polarity of a voltage we have a similar situation. The arrow of the the positive current direction has the same meaning as the signs + and – in the *positive voltage polarity* (see Figure 1.3.a).
If $V_2 > 0$ the left hand terminal has a higher potential than the right hand terminal. If $V_2 < 0$ the potential of the left hand terminal is smaller than that of the right hand terminal.
As has been already explained we can also denote a voltage between two terminals by two indices. This is beneficial in the case of big networks. The indices give the incident nodes. The first index is the positive terminal of the reference voltage (see Figure 1.3.b).
Thus we can omit the notation + and – and also the notation V_{AB} without difficulties.
Finally, if there is no confusion we can omit the double arrow (see Figure 1.3.c).
The relation between voltage and current in a conductor is given by *Ohm's law*:

$$V = RI, \tag{1.5}$$

in which R is supposed to be linear. R is called *resistance*. In Figure 1.4 it is shown how the positive voltage is related to the positive current.
We see that the positive current flows from plus to minus through the resistor. In other words we can say that *the current and the voltage are interrelated* (*belong to each other*).
The resistance of a wire with a constant cross-section is

$$R = \frac{\rho l}{A} = \frac{l}{\gamma A}. \tag{1.6}$$

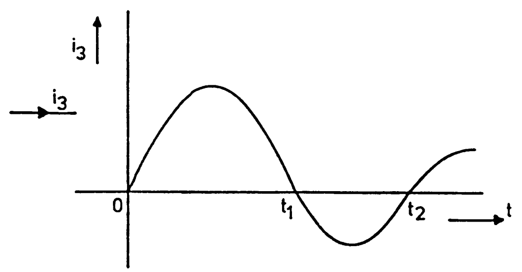


Figure 1.2

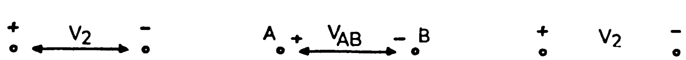


Figure 1.3 a b c

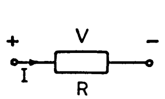


Figure 1.4

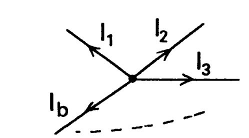


Figure 1.5

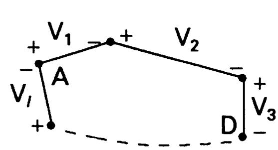


Figure 1.6

In this equation l is the length (m), A is the cross-section in square metres (m^2), ρ is the specific resistance (Ωm) and γ the specific conductance ($(\Omega m)^{-1}$). The values of ρ and γ depend on the material used. ρ and γ also depend on the temperature. This leads to non-linearity which we will discuss at the end of Chapter 10.

The *conductance* G is the inverse of the resistance:

$$G = \frac{1}{R}. \tag{1.7}$$

So Ohm's law can also be written as

$$I = GV. \tag{1.8}$$

We will assume that the connecting wires have no resistance, i.e. R = 0.

If R = 0 we speak of a *short circuit*; if G = 0 we speak of *open terminals*.

Kirchhoff's current law is:

$$\sum^{b} I_n = 0 \qquad n = 1,2,\ldots,b. \tag{1.9}$$

This equation refers to a node, where b wires (*branches*) coincide (see Figure 1.5). This formula also means that the sum of the currents flowing into the node equals the sum flowing out.

Positive currents are those which leave the node. *Kirchhoff's voltage law* is:

$$\sum_{m=1}^{l} V_m = 0 \qquad m = 1,2,\ldots,l. \tag{1.10}$$

This equation refers to a *loop* in which there are l voltages (see Figure 1.6).

Following the loop the drop in voltage is chosen as positive. The use of the node indications with figures or letters, together with the corresponding double voltage index results in a clear formula:

$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0.$$

So:

$$V_{AD} = V_{AB} + V_{BC} + V_{CD}. \tag{1.11}$$

Note the position of the indices in the left- and right-hand parts. Equation 1.11 is called the *index rule*. Not all nodes necessarily have a branch between them. The voltage law may also contain voltages of *fictitious* branches.

1.2 The voltage source and the current source

The voltage V_1 between the terminals of an *accumulator* with a varying *load* (see Figure 1.7) and the load current I_b have the idealised plot of Figure 1.8.

The analytic expression for $V_1 = f(I_b)$ is:

$$V_1 = V - R_i I_b. \tag{1.12}$$

R_i is the *internal resistance.*

With the aid of Kirchhoff's voltage law we can now draw the circuit of Figure 1.9.
I_S is the *short circuit current.* The element V is the (ideal) *voltage source.*
If we divide both terms of equation 1.12 by R_i we get

$$\frac{V_1}{R_i} = \frac{V}{R_i} - I_b \tag{1.13}$$

and together with Kirchhoff's current law this gives the network of Figure 1.10.

We have set $\frac{V}{R_i} = I$ and this is called the (ideal) *current source.*

As a collective name we will use the expression *source intensity* or *source strength* for a voltage source voltage and a current source current.

If we connect a resistor R to a voltage source V the current through that resistor is $I = \frac{V}{R}$ with $R \neq 0$. This leads to the important rule:

A voltage source must not be short-circuited.

If we connect a conductor G to a current source I the voltage across that conductor is $V = \frac{I}{G}$ with $G \neq 0$. This means:

The terminals of a current source must not be be opened.

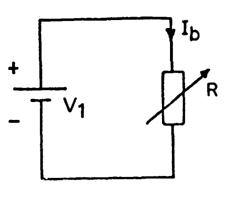


Figure 1.7

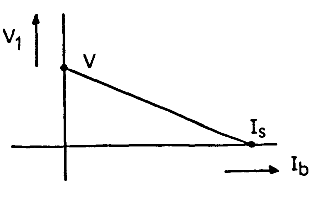


Figure 1.8

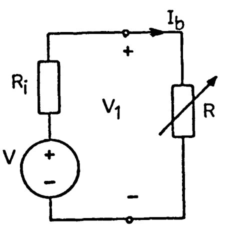


Figure 1.9

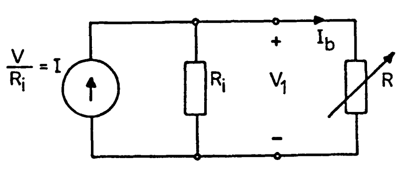


Figure 1.10

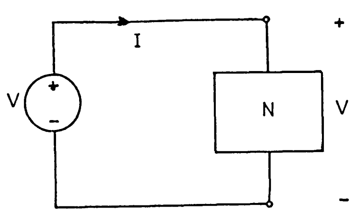


Figure 1.11

A voltage source with intensity zero is a *short circuit* and a current source with intensity zero is *open terminals*.

Two voltage sources in parallel are forbidden if the sum of the source strenghts is not zero (for in this case Kirchhoff's voltage law is not valid). Two voltage sources in parallel are not forbidden if the sum of the source strengths is zero. In that case, however, the source currents cannot be calculated.
Two current sources in series are forbidden if the sum of the source strengths is not zero (for in this case Kirchhoff's current law for the node between the two sources) is not valid. Two current sources in series are not forbidden if the sum of the source strengths is zero. In that case the source voltages cannot be calculated.

In network theory it often happens that there is analogy between two formulas, between two elements or between two circuits. For instance, one Kirchhoff's law turns into the other if one substitutes voltage for current and vice versa. We therefore say that the current law is *the dual* of the voltage law and vice versa.
The dual character is also found in

- voltage – current
- open nodes – short circuit
- resistance – conductance

In the following chapters we shall often meet this phenomenon of *duality*.

1.3 Energy and power

The voltage V_{AB} between two points A and B is defined as the work needed to move a unit charge (1 C concentrated in a point) from point B to point A.
If the charge is Δq the work is therefore

$$\Delta W = (V_A - V_B)\Delta q = V_{AB}\Delta q, \tag{1.14}$$

in which V_A and V_B are the potentials of the points A and B. If V_{AB} is constant (d.c.) and if the work is done in a time Δt, the average power is

$$P = \frac{\Delta W}{\Delta t} = V\frac{\Delta q}{\Delta t},$$

in which $V = V_{AB}$.
For $\Delta t \to 0$ we obtain

$$P = VI. \tag{1.15}$$

So the power, in the case of d.c., is the product of voltage and current.
Energy is expressed in joule (J), power in watt (W).
Power can be consumed or supplied. If a current I flows through a network N with two terminals (also called a *one-port*) and if the polarity of the voltage V is such that I flows from + to – the power consumed is positive (Figure 1.11).

The voltage source transports (positive) charge from minus to plus and so delivers electrical energy to N (this energy is supplied by the chemical or mechanical system outside the network).
The network N consumes this energy. We have pointed out before that the voltage and the current of a two-terminal network *belong to each other* if the current flows from plus to minus through the network. So we derive the following rule:

The energy $P = VI$ consumed is positive if current and voltage belong to each other.

We note that the current I does not necessarily have to be supplied by a voltage source; it can also be a current source or in general any other network.

1.4 Connection of resistors

One can easily show, using both Kirchhoff's laws and Ohm's law, that for the series connection (Figure 1.12) the total resistance is the sum of the separate resistances.

$$R = \sum_{k=1}^{n} R_k. \tag{1.16}$$

The dual situation is the parallel connection (see Figure 1.13). We find

$$G = \sum_{k=1}^{n} G_k. \tag{1.17}$$

1.5 Voltage and current division

A formula which is often used is the formula of voltage *division*. Consider the network of Figure 1.14. We calculate V_2, and find $I = \frac{V}{R_1 + R_2}$ and $V_2 = R_2 I$.
So

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V. \tag{1.18}$$

This is called *voltage division.*
The dual of voltage division is the *current division* (Figure 1.15). We find

$$V = \frac{I}{G_1 + G_2} \text{ and } I_2 = G_2 V.$$

So

$$I_2 = \frac{G_2}{G_1 + G_2} \cdot I. \tag{1.19}$$

Sometimes this formula is written with resistances. With $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$ we obtain

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I. \tag{1.20}$$

Note the index in the numerator.

1.6 The solution of larger networks

In general the solution of a network problem involves the computing of all voltages and currents in the network. As an example we shall solve the bridge circuit and attach a current to each branch (Figure 1.16).

Kirchhoff's current law for the nodes A, B, C and D gives:

$$I_1 = I_2 + I_5 \qquad \text{(a)}$$
$$I_2 = I_3 + I_4 \qquad \text{(b)}$$
$$I_1 = I_3 + I_6 \qquad \text{(c)}$$
$$I_6 = I_4 + I_5 \qquad \text{(d)}$$

Not all of these equations are independent. It follows from formulas (a) and (b) and from (c) and (d) that:

$$I_1 = I_3 + I_4 + I_5.$$

Kirchhoff's voltage law for the loops 1, 2 and 3 gives:

$$V_{AD} + V_{DC} + V_{CA} = 0 \qquad \text{(e)}$$
$$V_{AB} + V_{BD} + V_{DA} = 0 \qquad \text{(f)}$$
$$V_{BC} + V_{CD} + V_{DB} = 0 \qquad \text{(g)}$$

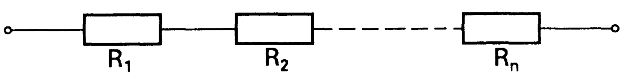


Figure 1.12

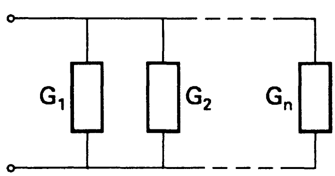


Figure 1.13

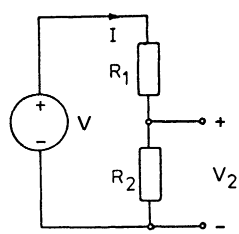


Figure 1.14

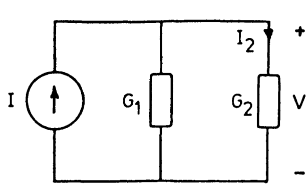


Figure 1.15

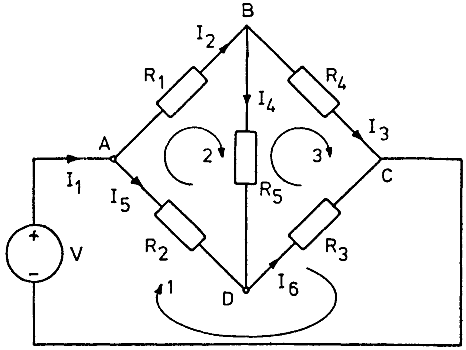


Figure 1.16

These three loops, each not enclosing smaller loops, are called *meshes*.
Kirchhoff's voltage law applied to the loop ABCD results in:

$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0. \qquad \text{(h)}$$

Formula (h) is dependent, because adding (f) and (g) yields (h).
Finally we apply Ohm's law:

$$V_{AB} = R_1I_2, \qquad \text{(i)}$$
$$V_{AD} = R_2I_5, \qquad \text{(j)}$$
$$V_{BD} = R_5I_4, \qquad \text{(k)}$$
$$V_{BC} = R_4I_3, \qquad \text{(l)}$$
$$V_{DC} = R_3I_6. \qquad \text{(m)}$$

The number of equations we obtain with this so-called *branch method* is not convenient and is also unnecessarily large. We shall now discuss two more systematic methods.

1.7 The mesh method

We again examine the bridge circuit and attach a so-called *mesh current* to each mesh. This is a fictitious current which *circulates* in a mesh (see Figure 1.17).
Consequently a *branch current* is the difference between two mesh currents if that branch is the separation of two meshes:

$$I_4 = I_2 - I_3. \qquad (1.21)$$

In an outer mesh the mesh current is equal to the branch current, e.g. I_1 in the source and I_2 in R_1. Now apply the voltage law to the three meshes. We write the intensity of the source in the left-hand part of the equation and the voltages across the resistors in the right-hand part:

$$V = R_2(I_1 - I_2) + R_3(I_1 - I_3)$$

or, in another sequence

$$V = (R_2 + R_3)I_1 - R_2I_2 - R_3I_3. \qquad (\alpha)$$

The first term in the right-hand part is the voltage created by the mesh current in the sum of the resistances in the mesh, the other terms are negative and are created by the 'counteracting' currents in the adjacent meshes.
Further note that the source voltage is positive, if, following the mesh direction, the plus terminal is left. In a similar way for the other meshes we find:

$$0 = -R_2I_1 + (R_1 + R_5 + R_2)I_2 - R_5R_3, \qquad (\beta)$$

$$0 = -R_3I_1 - R_5I_2 + (R_4 + R_3 + R_5)I_3. \qquad (\gamma)$$

With the three equations (α), (β) and (γ) we can solve the three mesh currents. All branch currents (and thus all branch voltages) are then known. In matrix notation the three mesh equations become: