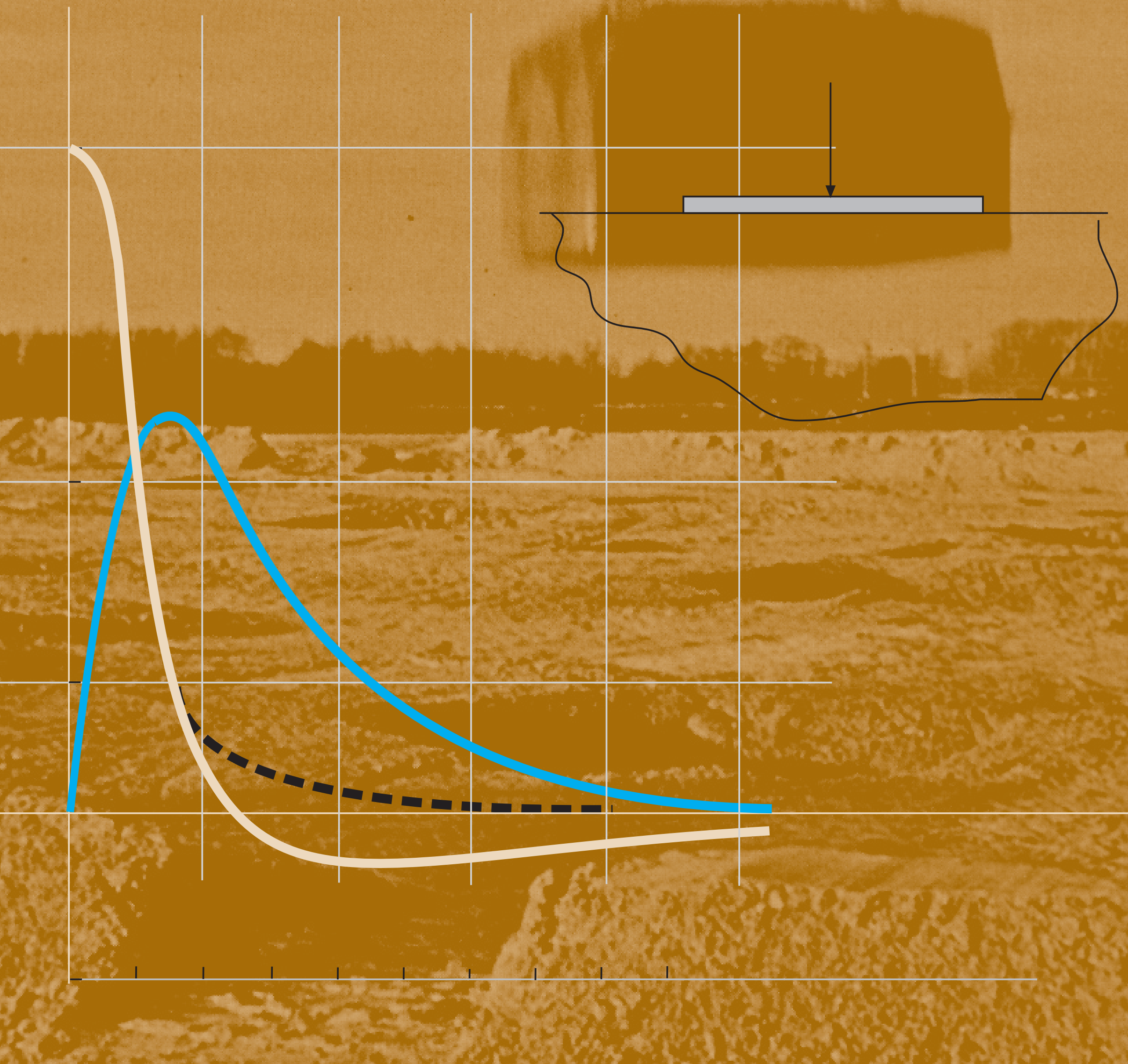
The vertical motion of foundations and pontoons



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Preface

This text collects the papers 'Vertical Vibration of Rigid Bodies on Deep Elastic Strata' and 'A Stoneley-Gibson-Varga Elastic Stratum' that have been published in the journal Heron (Volume 46, no. 1 (2001)).

The first chapter offers a survey of the vertical motion of rigid bodies resting on deep elastic strata. Four strata are distinguished:

- deep water,
- the homogeneous isotropic elastic half-space,
- the water saturated homogeneous isotropic porous elastic half-space and
- the Gibson half-space.

Four types of footings are considered: the strip, the circular disk and the embedded semi-cylinder and hemi-sphere.
In particular attention has been given to the distinction between compressible and incompressible strata, and to the distinction between low and high frequency factors of the oscillatory motion.

The second chapter provides a geometrically non-linear generalization of the Gibson soil. Some remarkable solutions concerning excavations and indented rigid punches are presented.
The results provide a first approximation of the behaviour of foundations on real soils in the case of small soil strains.

The author is indebted to Prof dr Ton Vrouwenvelder of Delft University of Technology and to Jacques Schievink, publisher of text books at VSSD. They provided indispensible encouragement, helpful criticism, cooperation and assistance in preparing the final text and illustrations.

Voorburg, November 2002

Contents

1. A comparative treatise on the vertical vibration of rigid bodies on deep elastic strata

Deep water, Gibson soil, Homogeneous (water saturated porous) isotropic elastic half-space
Compressible versus incompressible strata
Low frequency versus high frequency factors

Keywords: **lumped parameters, dynamic subgrade reaction, floating bodies, resting footings.**

1.1. Introduction

In soil mechanics a Gibson soil is defined as being an incompressible, isotropic, elastic half-space $X_1 \geq 0$ in which the shear modulus μ increases linearly with depth X_1 from zero on the upper surface $X_1 = 0$, according to the equation $\mu = mX_1$ with m a positive constant.

In his famous 1967-paper Gibson showed that the upper surface of this elastic deep stratum reacts under static normal loading like a uniform bed of springs, a so-called Winkler foundation (Gibson (1967)).

Any point of the upper-surface $X_1 = 0$ settles an amount w $(X_1 = 0)$, directly proportional to the local intensity $-q(X_1 = 0)$ of applied normal stress according to the law $w(X_1 = 0) = q(X_1 = 0)/(2m)$; outside the loaded area the upper-surface does not settle.

It has been noticed that 1) the induced deformation at the locations $X_1 > 0$ is irrotational, 2) the state of stress on the location of the (loaded) upper surface $X_1 = 0$ is purely isotropic and 3) the settlement $w(X_1)$ of a point at the level $X_1 > 0$ is directly proportional to the all-round pressure $-p(X_1)$ at that point according to $w(X_1) = p(X_1)/(2m)$. These settlements at the levels $X_1 > 0$ decrease with increasing horizontal or vertical distances from the loaded surface area. Of course, the state of stress on the planes $X_1 > 0$ is not isotropic since the Gibson soil possesses shear rigidity at planes $X_1 > 0$. Further, it has been realized (Lekhnitski (1962) and Gibson (1967)) that the stress distribution in the Gibson soil due to a static normal surface loading corresponds exactly to the stress distribution in an incompressible homogeneous isotropic elastic half-space due to the same surface loading.

When it is assumed that the Gibson soil has been subjected initially to an hydrostatic stress distribution due to its self-weight own weight, the quantity $2m$ must be replaced by $(\rho g+2m)$ with ρ the uniform mass density and g the acceleration due to gravity.

It has been shown that the linear equations governing the dynamics of the Gibson soil resemble mathematically the linearized equations of the deep water motion (Kruijtzer (1976)). For example, when we analyse a group of plane harmonic waves of wave length $\lambda/2\pi\kappa$ travelling through the Gibson soil in a horizontal direction with velocity c, it is found that there exists irrotanional surface waves with wave velocity $c = ((\rho g + 2m)/(\rho\kappa))^{1/2}$. These irrotational waves are mathematically similar to the (irrotational) gravitational deep water surface waves with velocity $c = (g/\kappa)^{1/2}$.

In a recent paper we have introduced the geometrically non-linear Gibson soil, the so-called Stoneley-Gibson-Varga elastic half-space, in which the actual strees and the left stretch tensor are correlated (Kruijtzer (2001)). It was shown that the non-linear equations of the irrotaional dynamics of this half-space resemble mathematically the classical non-linear equations of the irrotanional deep water motion. Furthermore, it was shown that the settlement $w(X_1)$ of a point at the level $X_1 > 0$ is directly proportional to an all-round pressure $-p(X_1)$ at that point according to $w(X_1) = p(X_1)/(\rho g + 2m)$. These settlements at the levels $X_1 > 0$ decrease with increasing horizontal or vertical distances from the loaded surface area. Of course, the state of stress on the planes $X_1 > 0$ is not isotropic since the Gibson soil possesses shear rigidity at planes $X_1 > 0$. It may be noticed that the static stress distribution in the Stoneley-Gibson-Varga elastic half-space is not similar to the stress distribution in the corresponding geometrically non-linear incompressible homogeneous elastic half-space.

In this treatise we compare the responses of deep water, the Gibson soil and the homogeneous isotropic elastic half-space on low and high frequency vertical surface loadings including the effects of compressibility and incompressibility of these strata. Our comparative treatise reveals not only various mathematical and physical resemblences or similarities,but also provides with an application in soil mchanics.

In theoretical soil mechanics water saturated soils are often conceived to behave like water saturated porous elastic strata (elastic skeletons). In the *fully drained* state there is no excess of water pressure so that the stratum behaves like an ordinary elastic medium. In the *fully undrained* state the water velocity equals the solid velocity. In this case the medium behaves as being an elastic medium but the modulus of compression of the medium depends mainly on the elasticity of water volume and scantly on the bulkmodulus of the elastic skeleton (ensemble of packed grains).

The fundamental 1956-paper of G. de Josselin de Jong is our guide in considering the corresponding responses of a water saturated porous isotropic elastic half-space.

Finally we notice that the truncated semi-infinite cone-model of the elastic half-space for vertical vibrations (J.P.Wolf (1985)) is based on the results of the classical half-space theory.

1.2. Motion of footings and floating bodies

Linear dynamics

We want to compare the vertical motion of a rigid footing resting on the surface of a Gibson soil or a homogeneous incompressible isotropic deep elastic stratum with the corresponding heave motion of a rigid floating body on deep water (Figure 1.1). We did not find non-linear solutions, so we restrict ourselves necessarily to solutions of the linearised equations of motion. In the linearised theories the difference between the gradients with respect to the material coordinates and the gradients with respect to the spatial coordinates is disregarded. Further, particular stress distributions due to special external loadings may be superposed.

In general, the behavior of linear systems can be described in either the time domain or the frequency domain. It may be well-known that the characteristics of the high frequency steady state motion of such systems due to external periodic loading may be obtained from the initial motion of these systems due to an impulsive external loading (Maskell and Ursell (1970) in Ursell (1994), Cummings (1962) and Ogilvie (1964)).

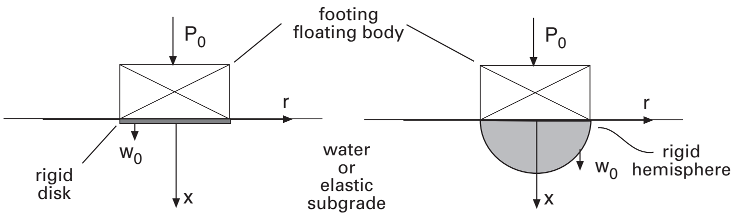

Figure 1.1. Footing on an elastic subgrade or floating body on deep water.

Single degree of freedom model

In engineering mechanics the vertical motion of footings and floating bodies is often described by a lumped-parameter-system with a single-degree-of-freedom. With $w_0 = w_0(t)$ the vertical displacement of the rigid body and $P_0 = P_0(t)$ the external vertical centric loading the equation of motion of the body reads

$$(M^{\circ} + \overline{M}^{*})\ddot{w}_0(t) + \overline{C}\dot{w}_0(t) + \overline{K}w(t) = P_0(t) \tag{1.2.1}$$

with M° the footing mass, $\overline{M}^{*}$ the added (in-phase) subgrade mass, $\overline{C}$ the coefficient of damping due to wave radiation through the subgrade, and $\overline{K}$ the coefficient of the subgrade restoring force.

The value of $\overline{M}^{*}$, $\overline{C}$ and $\overline{K}$ depend on the induced subgrade motion.

With $P_0(t) = \hat{P}_0 e^{i\omega t}$ and $w_0(t) = \hat{w}_0 e^{i\omega t}$ the steady state motion of the linear system is given by the equation

$$-(M^{\circ} + M^{*}(\omega))\omega^2 \hat{w}_0 + i\omega\, C(\omega)\hat{w}_0 + K(\omega)\hat{w}_0 = \hat{P}_0 \tag{1.2.2}$$

with $M^{*}(\omega)$ the added mass, $C(\omega)$ the damping factor and $K(\omega)$ the restoring coefficient. These quantities, $M^{*}(\omega)$, $C(\omega)$ and $K(\omega)$ are frequency dependent.

We consider the cases in which a weightless rigid circular disk or infinitely long rigid strip, an embedded rigid hemisphere or infinitely long semi-cylinder are attached to the upper surface of the subgrade (Figure 1.1) and are loaded by a vertical centric force $P_0 = P_0(t)$, so that the rigid bases undergo a vertical displacement $w_0 = w_0(t)$ according to

$$w_0(t) = F(t)\, P_0(t) \tag{1.2.3}$$

with $F(t)$ the response function. If $P_0(t) = \hat{P}_0\, e^{i\omega t}$ and $w_0(t) = \hat{w}_0\, e^{i\omega t}$ then the equation (1.2.3) of the steady state motion takes the form

$$\hat{w}_0\, e^{i\omega t} = -\,(f_1 + \mathrm{i}f_2)\ \hat{P}_0\, e^{i\omega t} \tag{1.2.4}$$

with f_1 and f_2 two functions of frequency being effectively the in-phase and out-phase components of the displacement $w(t)$. In Appendix 1.1 the results of Bycroft are schematically presented and completed.

Deep water

The incompressible inviscid fluid subgrade is assumed to be intially, at time $t = 0$, at a state of rest. We suppose that at time $t = 0_+$ the upper surface has been subjected to an impulsive pressure $-p_0(t = 0_+)$. According to the linearized theory this impulsive pressure is represented by the value $-\rho\varphi_0(t = 0_+)$, where φ_0 is the value of the velocity potential at the loaded area at time $t = 0_+$ (Stoker (1957), p. 150, Lamb (1945), p. 384):

$$p_0(r,t = 0_+) = -\rho\varphi_0(r,t = 0_+) \tag{1.2.5a}$$

with ρ the water mass density and r the radius of the loaded area. (We may notice that a permanent flow passing an obstacle generates at each instant of time an impulsive pressure on the obstacle).

We now consider the case in which a thin weightless rigid circular disk with radius r_0 at the upper surface is loaded by a vertical and centric impulsive force $\tilde{P}_0$. Then (Lamb (1945), p. 120 and p. 138)

$$\begin{aligned} p_0(r,t{=}0_+) &= -\rho\frac{2}{\pi}\cdot\ \dot{w}_0(t = 0_+)\cdot(r_0^2 - r^2)^{1/2} \quad (0 \le r < r_0) \\ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad (r > r_0) \end{aligned} \tag{1.2.5b}$$

where $\dot{w}_0\ (t = 0_+)$ is the initial velocity of the disk.

Integration of (1.2.5b) with respect to r gives

$$\tilde{P}_0 = \int_0^{r_0} p_0\ 2\pi r\ dr = \frac{4}{3}\ \rho r_0^3\ \dot{w}_0(t = 0_+) \tag{1.2.5c}$$

It follows that high frequency added mass M^* is given by $M^* = (4/3)\rho r_0^3$.

In the case of an infinitely long weightless rigid strip of width $2d_0$ the added mass M^* is given by $M^* = (1/2)\pi\rho d_0^2$ (Lamb (1945), p. 85).

In the theory of linearized ship motion the rigid disk is replaced by a semi-submerged sphere and the rigid strip is replaced by a semi-submerged infinitely long cylinder. The vertical impulsive force $\tilde{P}_0$ on a floating sphere of radius r_0 and with mass $M° = (2/3)\pi\rho r_0^3$ gives rise to the high frequency added mass $M^* = M°/2 = (1/3)\pi\rho r_0^3$. The vertical impulsive line-force on a floating cylinder of mass $M° = (1/2)\pi\rho r_0^2$ per unit length, gives rise to the high frequency added mass $M^* = M°$ (Ursell (1994)), Lamb (1945), pp. 80 and 124). These reults may be proved as follows.

We consider the translational motion of weightless rigid sphere in an infinite incompressible inviscid fluid. At each instant of time the moving sphere generates an instantaneous fluid motion. The fluid pressure contains a linear portion and a non-linear portion (Lamb (1945), p. 124). The non-linear portion generates a zero resulting force on the sphere. At the location of a plane through the center of the sphere perpendicular to the direction of motion of the sphere, the linear portion of the pressure vanishes while the non-linear portion does not vanish. The kinetic energy of the fluid in given by $(2/3)\pi\rho r_0^3\,\dot{w}_0^2$ with ρ the fluid mass density,r_0 the radius of the sphere and $\dot{w}_0$ the velocity of the sphere. The resultant effect of the fluid pressure in the direction of the motion is given by $-(2/3)\pi\rho r_0^3\,\ddot{w}_0$, so that $M^* = (2/3)\pi\rho r_0^3$ is the added in-phase mass. Since according to the linearised theory at the plane through the center of the sphere and pendicular in the direction of the motion of the sphere the pressure vanishes, we are lead into the following result.

When a rigid floating hemisphere of radius r_0 is subjected to a vertical centric impulsive force $\tilde{P}_0$ at time $t = 0$ (Figure 1.1) the sphere gets the downward initial velocity $w_0(t = 0_+)$ according to

$$\tilde{P}_0 = (M^\circ + M^*)\, w_0(t = 0_+) \qquad (1.2.6)$$

with M° is the mass of the semi-sphere and M^* is the added in-phase fluid mass: $M^\circ = (2/3)\pi\rho r_0^3$ and $M^* = (1/3)\pi\rho r_0^3 = M^\circ/2$. In the case of an infinitely long circular semi-cylinder $M^\circ = M^* = (1/2)\pi\rho\, r_0^2$ (Lamb (1945), p. 77).

The analysis by Newman (1969) gives rise to a quantification of the high frequency added mass and to an indication of the low frequency added mass as follows.

The forced motion of a floating body resting on deep water generates surface waves due to gravity. On the free surface the pressure $-p_0$ is equal to zero, so that at this location the linearised condition $\omega^2\varphi - g\,\dot{w} = 0$ must be satisfied, with φ the velocity potential, $\dot{w}$ the vertical surface velocity and g the acceleration due to gravity (Lamb (1945), p. 363).

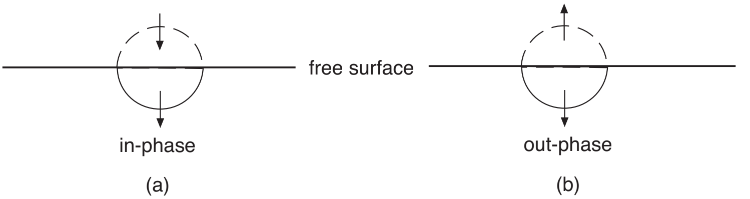

Figure 1.2. Method of images.

In the case of very high frequencies $\omega^2\varphi >> g\dot{w}$, so that the potential φ vanishes on the free surface (short waves). By the 'method of images' (Figure 1.2a) an upper hemisphere can be added to the lower hemisphere. When the upper and lower hemisphere move in-phase, the normal velocities on the upper and lower hemisphere are opposite in sign, so that the potential φ is equal to zero on the 'free surface'. This corresponds indeed to the problem of a sphere moving in a fluid of infinite extent, so that the high frequency mass added to the hemisphere is equal to $M^* = (\pi/3)\rho r_0^3 = M^\circ/2$.

In the case of very low frequencies ω we have $\omega^2\varphi << g\dot{w}$ on the free surface so that the normal surface velocity vanishes (long waves). When an image hemisphere is added to the submerged hemisphere and the upper and lower hemisphere move 180° ot of phase (Figure 1.2b), the vertical velocity at the original free surface is zero. In fact there is no longer a free surface problem, but the problem of pulsation of a dilitating 'sphere' of changing volume. From the litterature on ship motions it appears that the low frequency added mass is equal to about (3/2) times the high frequency added mass (compare the purely radial expansion motion of a sphere in a infinite incompressible fluid in which case φ does not vanish at the 'free surface' and the added mass is equal to $4{\cdot}(3/2)(2/3)\pi\rho r_0^3$ (Lamb (1945), p. 122)., but is smaller than twice the high frequency mass (Appendix 1.3). It is noticed that in two-dimensional problems the low frequency added mass becomes mathematically infinite, because from the continuity of finite flux of fluid, oscillating back and forth, there is only the way out at infinity. In three dimensions this infinity does not occur because the fluid flux can distribute itself spatially in all three directions. On the