Fracture Mechanics

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2nd Edition

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Preface to the First Edition

While teaching a course on fracture mechanics at Delft University of Technology we discovered that although there are a few excellent textbooks, their subject matter covers developments only up to the early 1970s. Consequently there was no systematic treatment of the concepts of elastic-plastic fracture mechanics. Also the description of fracture mechanics characterisation of crack growth needed updating, especially for sustained load fracture and unstable dynamic crack growth.

In the present textbook we have attempted to cover the basic concepts of fracture mechanics for both the linear elastic and elastic-plastic regimes, and three chapters are devoted to the fracture mechanics characterisation of crack growth (fatigue crack growth, sustained load fracture and dynamic crack growth).

There are also two chapters concerning mechanisms of fracture and the ways in which actual material behaviour influences the fracture mechanics characterisation of crack growth. The reader will find that this last topic is treated to some way beyond that of a basic course. This is because to our knowledge there is no reference work that systematically covers it. A consequence for instructors is that they must be selective here. However, any inconvenience thereby entailed is, we feel, outweighed by the importance of the subject matter.

This textbook is intended primarily for engineering students. We hope it will be useful to practising engineers as well, since it provides the background to several new design methods, criteria for material selection and guidelines for acceptance of weld defects.

Many people helped us during preparation of the manuscript. We wish to thank particularly J. Zuidema, who made vital contributions to uniform treatment of the energy balance approach for both the linear elastic and elastic-plastic regimes; R.A.H. Edwards, who assisted with the chapter on sustained load fracture; A.C.F. Hagedorn, who drew the figures for the first seven chapters; and the team of the VSSD, our publisher, whose patience was sorely tried but who remained unbelievably co-operative.

Finally, we wish to thank the National Aerospace Laboratory NLR and the Boiler and Pressure Vessel Authority 'Dienst voor het Stoomwezen' for providing us the opportunity to finish this book, which was begun at the Delft University of Technology.

H.L. Ewalds
R.J.H. Wanhill

September 1983

Preface to the Second Edition

In 1991, the fifth reprint of the first edition of the textbook “Fracture Mechanics”, by H.L. Ewalds and R.J.H. Wanhill, was published. Obviously the field of fracture mechanics has developed further since that time. A new edition was needed. The task fell mainly to the new authors, M. Janssen and J. Zuidema, both in the Department of Materials Science at Delft University of Technology, with assistance by R.J.H. Wanhill, of the National Aerospace Laboratory NLR. The original first author, H.L. Ewalds, indicated that he no longer wished to be involved with this textbook. We respect his decision, and thank him for his major contribution to the First Edition, which has been very successful.

This second edition is the result of numerous revisions, updates and additions. These were driven by the ongoing development of fracture mechanics, but also by teaching the course on fracture mechanics at Delft University of Technology. The fracture mechanics parameters K, G and J are now treated in a more basic manner. Test methods for J_{Ic} and for crack arrest toughness are updated. The development of failure assessment based on elastic-plastic fracture mechanics is reflected in a comprehensive treatment. On the subject of subcritical crack growth more attention is paid to the important topic of the initiation and growth of short fatigue cracks.

Throughout the book some paragraphs are typeset in a smaller font. This text is intended to provide additional background information on certain subjects, but is not considered essential for a basic understanding.

We would like to acknowledge the assistance of colleagues in preparing this second edition. With critical reading and profound discussions A.R. Wachters helped considerably in drawing up the part on the J integral. G. Pape did the preparatory work necessary for updating the chapter on dynamic fracture. A. Bakker contributed to the treatment of the R6 failure assessment procedure. Finally, A.H.M. Krom provided useful comments and suggestions on various subjects.

The authors wish to thank our publisher, J.E. Schievink of the VSSD, for his encouragement and co-operation in creating this new edition.

M. Janssen
J. Zuidema
R.J.H. Wanhill

March 2002

Part I
Introduction

1 An Overview

1.1 About this Course

This course is intended as a basic grounding in fracture mechanics for engineering use. In order to compile the course we have consulted several textbooks and numerous research articles. In particular, the following books have been most informative and are recommended for additional reading:

- D. Broek, "Elementary Engineering Fracture Mechanics", Martinus Nijhoff (1986) The Hague;
- J.F. Knott, "Fundamentals of Fracture Mechanics", Butterworths (1973) London;
- Richard W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", John Wiley and Sons (1988) New York;
- T.L. Anderson, "Fracture Mechanics, Fundamentals and Applications", CRC Press (1991) Boston.

Four international journals are also recommended:

- Fatigue and Fracture of Engineering Materials and Structures;
- International Journal of Fatigue;
- International Journal of Fracture;
- Engineering Fracture Mechanics.

As indicated in the table of contents the course has been divided into five parts. Part I, consisting of this chapter, is introductory. In Part II the well established subject of Linear Elastic Fracture Mechanics (LEFM) is treated, and this is followed in Part III by the more recent and still evolving topic of Elastic-Plastic Fracture Mechanics (EPFM). In Part IV the applicability of fracture mechanics concepts to crack growth behaviour is discussed: namely subcritical, stable crack growth under cyclic loading (fatigue) or sustained load, and dynamic crack growth beyond instability. Finally, in Part V the mechanisms of fracture in actual materials are described together with the influence of material behaviour on fracture mechanics-related properties.

1.2 Historical Review

Strength failures of load bearing structures can be either of the yielding-dominant or fracture-dominant types. Defects are important for both types of failure, but those of primary importance to fracture differ in an extreme way from those influencing yielding and the resistance to plastic flow. These differences are illustrated schematically in figure 1.1.

For yielding-dominant failures the significant defects are those which tend to warp and interrupt the crystal lattice planes, thus interfering with dislocation glide and providing a resistance to plastic deformation that is essential to the strength of high strength metals. Examples of such defects are interstitial and out-of-size substitutional atoms, grain boundaries, coherent precipitates and dislocation networks. Larger defects like inclusions, porosity, surface scratches and small cracks may influence the effective net section bearing the load, but otherwise have little effect on resistance to yielding.

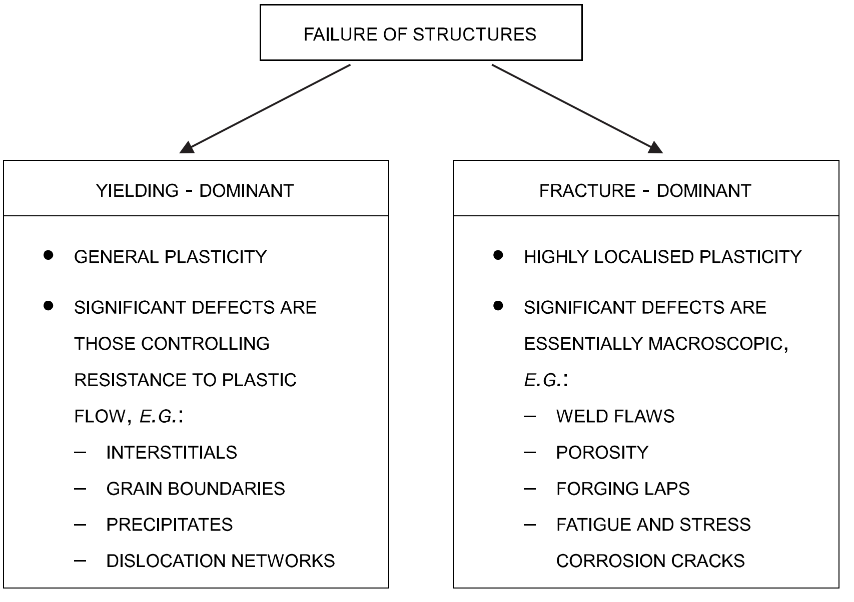

Figure 1.1. Types of structural failure.

For fracture-dominant failures, *i.e.* fracture before general yielding of the net section, the size scale of the defects which are of major significance is essentially macroscopic, since general plasticity is not involved but only the local stress-strain fields associated with the defects. The minute lattice-related defects which control resistance to plastic flow are not of direct concern. They are important insofar as the resistance to plastic flow is related to the material's susceptibility to fracture.

Fracture mechanics, which is the subject of this course, is concerned almost entirely with fracture-dominant failure. The commonly accepted first successful analysis of a fracture-dominant problem was that of Griffith in 1920, who considered the propagation of brittle cracks in glass. Griffith formulated the now well-known concept that an existing crack will propagate if thereby the total energy of the system is lowered, and he assumed that there is a simple energy balance, consisting of a decrease in elastic strain energy within the stressed body as the crack extends, counteracted by the energy needed to create the new crack surfaces. His theory allows the estimation of the theoretical strength of brittle solids and also gives the correct relationship between fracture strength and defect size.

The Griffith concept was first related to brittle fracture of metallic materials by Zener and Hollomon in 1944. Soon after, Irwin pointed out that the Griffith-type energy balance must be between (i) the stored strain energy and (ii) the surface energy plus the work done in plastic deformation. Irwin defined the 'energy release rate' or 'crack driving force', G, as the total energy that is released during cracking per unit increase in crack size. He also recognised that for relatively ductile materials the energy required to form new crack surfaces is generally insignificant compared to the work done in plastic deformation.

In the middle 1950s Irwin contributed another major advance by showing that the energy approach is equivalent to a stress intensity (K) approach, according to which fracture occurs when a critical stress distribution ahead of the crack tip is reached. The material property governing fracture may therefore be stated as a critical stress intensity, K_c, or in terms of energy as a critical value G_c.

Demonstration of the equivalence of G and K provided the basis for development of the discipline of Linear Elastic Fracture Mechanics (LEFM). This is because the form of the stress distribution around and close to a crack tip is always the same. Thus tests on suitably shaped and loaded specimens to determine K_c make it possible to determine what cracks or crack-like flaws are tolerable in an actual structure under given conditions. Furthermore, materials can be compared as to their utility in situations where fracture is possible. It has also been found that the sensitivity of structures to subcritical cracking such as fatigue crack growth and stress corrosion can, to some extent, be predicted on the basis of tests using the stress intensity approach.

The beginnings of Elastic-Plastic Fracture Mechanics (EPFM) can be traced to fairly early in the development of LEFM, notably Wells' work on Crack Opening Displacement (COD), which was published in 1961. In 1968 Rice introduced an elastic-plastic fracture parameter with a more theoretical basis: the J integral. Although both COD and J are now well established concepts, EPFM is still very much an evolving discipline. The reason is the greater complexity of elastic-plastic analyses. Important topics are:

- the description of stable ductile crack growth (tearing),
- the development of failure assessment methods that combine the effects of plasticity and fracture.

As opposed to using the above-mentioned global fracture mechanics parameters, fracture problems are also increasingly being tackled by means of local fracture criteria. Here the mechanical conditions that actually exist in the crack tip region are being determined and are being related to the material properties.

1.3 The Significance of Fracture Mechanics

In the nineteenth century the Industrial Revolution resulted in an enormous increase in the use of metals (mainly irons and steels) for structural applications. Unfortunately, there also occurred many accidents, with loss of life, owing to failure of these structures. In particular, there were numerous accidents involving steam boiler explosions and railway equipment.

Some of these accidents were due to poor design, but it was also gradually discovered that material deficiencies in the form of pre-existing flaws could initiate cracking and fracture. Prevention of such flaws by better production methods reduced the number of failures to more acceptable levels.

A new era of accident-prone structures was ushered in by the advent of all-welded designs, notably the Liberty ships and T-2 tankers of World War II. Out of 2500 Liberty ships built during the war, 145 broke in two and almost 700 experienced serious failures. Many bridges and other structures also failed. The failures often occurred under very low stresses, for example even when a ship was docked, and this anomaly led to extensive investigations which revealed that the fractures were brittle and that flaws and stress concentrations were responsible. It was also discovered that brittle fracture in the types of steel used was promoted by low temperatures. This is depicted in figure 1.2: above a certain transition temperature the steels behave in a ductile manner and the energy required for fracture increases greatly.

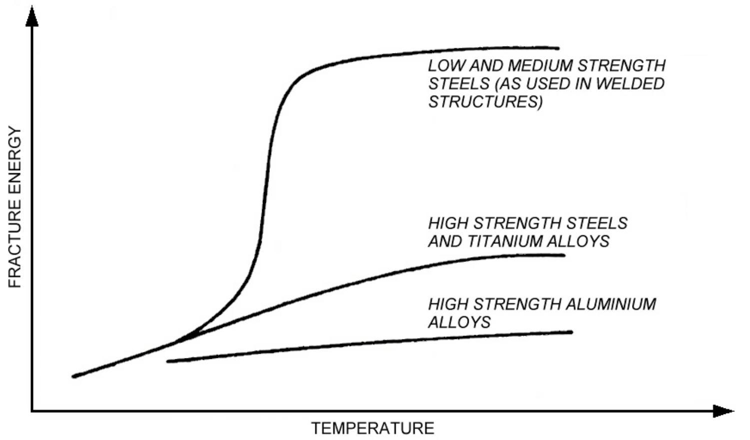

Figure 1.2. Schematic of the general effect of temperature on the fracture energy of structural metals.

Current manufacturing and design procedures can prevent the intrinsically brittle fracture of welded steel structures by ensuring that the material has a suitably low transition temperature and that the welding process does not raise it. Nevertheless, service-induced embrittlement, for example hydrogen embrittlement in the petrochemical industries, irradiation effects in nuclear pressure vessels and corrosion fatigue in offshore platforms, remains a cause for concern.

Looking at the present situation it may be seen from figure 1.3 that since World War II the use of high strength materials for structural applications has greatly increased.

These materials are often selected to obtain weight savings — aircraft structures are an obvious example. Additional weight savings have come from refinements in stress analysis, which have enabled design allowables to be raised. However, it was not recognised until towards the end of the 1950s that although these materials are not intrinsi-

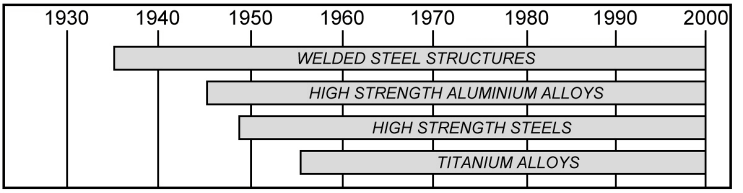

Figure 1.3. Introduction of high strength materials for structural applications.

cally brittle, the energy required for fracture is comparatively low, as figure 1.2 shows. The possibility, and indeed occurrence, of this low energy fracture in high strength materials stimulated the modern development of fracture mechanics.

The object of fracture mechanics is to provide quantitative answers to specific problems concerning cracks in structures. As an illustration, consider a structure containing pre-existing flaws and/or in which cracks initiate in service. The cracks may grow with time owing to various causes (for example fatigue, stress corrosion, creep) and will generally grow progressively faster, figure 1.4.a. The residual strength of the structure, which is the failure strength as a function of crack size, decreases with increasing crack size, as shown in figure 1.4.b. After a time the residual strength becomes so low that the structure may fail in service.

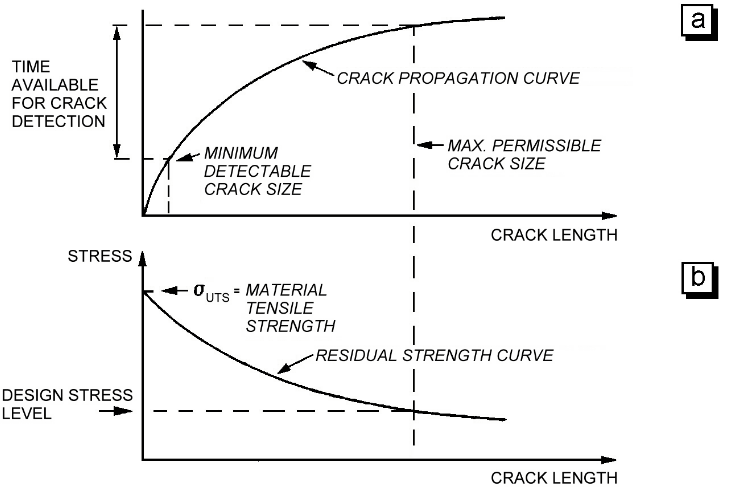

Figure 1.4. The engineering problem of a crack in a structure.

With respect to figure 1.4 fracture mechanics should attempt to provide quantitative answers to the following questions:

1. What is the residual strength as a function of crack size?
2. What crack size can be tolerated under service loading, *i.e.* what is the maximum permissible crack size?
3. How long does it take for a crack to grow from a certain initial size, for example the minimum detectable crack size, to the maximum permissible crack size?

4. What is the service life of a structure when a crack-like flaw (*e.g.* a manufacturing defect) with a certain size is assumed to exist?
5. During the period available for crack detection how often should the structure be inspected for cracks?

This course is intended to show how fracture mechanics concepts can be applied so that these questions can be answered.

In the remaining sections 1.4 – 1.11 of this introductory chapter an overview of the basic concepts and applications of LEFM and EPFM are given in preparation for more detailed treatment in subsequent chapters.

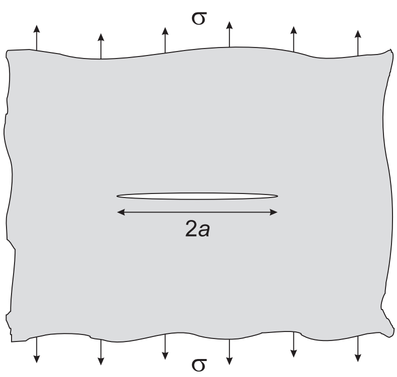

Figure 1.5. A through-thickness crack in a loaded infinite plate.

1.4 The Griffith Energy Balance Approach

Consider an *infinite* plate that is subjected to a uniform tensile stress, σ, applied at infinity (see figure 1.5). Suppose that we introduce a through-thickness crack of length $2a$. In the area directly above and below the crack the stress (in the loading direction) will decrease significantly and will even become zero along the crack flanks. Hence introduction of the crack changes the elastic strain energy stored in the plate. We can roughly estimate this change by assuming that in a circle-shaped area of radius a around the crack the stress has become zero, while the remainder of the plate experiences the same stress as before. In this case the elastic energy in the plate has decreased by an amount equal to the volume of the stress-free material times the original elastic energy per unit volume, *i.e.* ½×stress×strain. Assuming linear elastic material behaviour, *i.e.* a Young's modulus E, the elastic energy change would be[1]:

$$\pi a^2 \cdot \frac{\sigma^2}{2E} = \frac{1}{2}\frac{\pi\sigma^2 a^2}{E} \ . \qquad (1.1)$$

Obviously, this is only an approximation because the stress field becomes non-

[1] In this section we consider two-dimensional geometries only and all energies and forces are defined per unit thickness.

homogeneous near the crack, as will be shown in chapter 2. Griffith used a stress analysis developed by Inglis to show that for an infinite plate the elastic energy change is actually given by

$$U_a = -\frac{\pi\sigma^2 a^2}{E} , \tag{1.2}$$

where U_a = change in the elastic strain energy of the plate caused by introducing a crack with length $2a$. The minus sign shows this change is a decrease in elastic energy.

The introduction of a crack will require a certain amount of energy. Griffith assumed that for ideally brittle materials this is in the form of surface energy. A crack with length $2a$ in a plate involves the creation of a crack surface area (defined per unit thickness) equal to $2\cdot(2a) = 4a$, leading to an increase in surface energy of

$$U_\gamma = 4a\cdot\gamma_e , \tag{1.3}$$

where U_γ = change in surface energy of the plate due to introduction of a crack with length $2a$,
γ_e = surface energy per unit area, *i.e.* the surface tension.

Griffith postulated that a crack will extend when the *potential* energy decreases. He considered the surface energy as a part of this potential energy. In practice the energy involved in creating crack surfaces will not be reversible due to several reasons (oxidation etc.) and strictly speaking is not part of the potential energy. However, as long as only growing cracks are considered, the irreversibility of the surface energy is not relevant. Here, the potential energy according to Griffith will be referred to as the *total* energy.

For a real plate, *i.e.* one with finite dimensions, the total energy U is that of the plate *and* its loading system. When a crack is present the total energy U is composed of

$$U = U_o + U_a + U_\gamma - F , \tag{1.4}$$

where U_o = total energy of the plate *and* its loading system before introducing a crack (a constant),
F = work performed by the loading system during the introduction of the crack = load × displacement.

The combination of plate and loading system is assumed to be isolated from its surroundings, *i.e.* no work is performed on the plate or on the loading system from outside. This explains why F must be subtracted in equation (1.4): if the loading system performs work it goes at the expense of the energy content of the loading system and therefore lowers the total energy U. A more extensive treatment will be given in section 4.2.

In this introductory chapter we will conveniently assume that no work is done by the loading system. This is the case if the specimen is loaded by a constant displacement, a so-called *fixed grip* condition. Then the term F in equation (1.4) will vanish. Introducing a crack now leads to a decrease in elastic strain energy of the plate, *i.e.* U_a is negative, because the plate loses stiffness and the load applied by the fixed grips will drop. A plate with finite dimensions resembles an infinite plate when $2a << W$, the plate width. Consequently, the total energy U of a finite plate loaded with fixed grips and containing a small crack is approximately

$$U = U_o + U_a + U_\gamma = U_o - \frac{\pi\sigma^2 a^2}{E} + 4a\gamma_e \,. \qquad (1.5)$$

Following Griffith, crack extension will occur when U decreases. In order to formulate a criterion for crack extension, we consider an increase of the crack length by d(2a). Since U_o is constant, it will not change and ${}^{dU_o}/_{d(2a)}$ is zero. Also, since no work is done by the loading system, the driving force for crack extension can be delivered only by the decrease in elastic energy dU_a due to the crack length increase d(2a). The crack will extend when the *available* energy dU_a is larger than the energy *required* dU_γ. Thus the criterion for crack extension is

$$\frac{dU}{d(2a)} = \frac{d}{d(2a)}(U_a + U_\gamma) < 0 \quad \text{or} \quad \frac{d}{d(2a)}\left(-\frac{\pi\sigma^2 a^2}{E} + 4a\gamma_e\right) < 0 \,. \qquad (1.6)$$

This is illustrated in figure 1.6. Figure 1.6.a schematically represents the two energy terms in equation (1.6) and their sum as functions of the introduced crack length, 2a. Figure 1.6.b represents the derivative, ${}^{dU}/_{d(2a)}$. When the elastic energy release due to a potential increment of crack growth, d(2a), outweighs the demand for surface energy for the same crack growth, the introduction of a crack will lead to its unstable propagation.

From the criterion for crack extension, equation (1.6), one obtains

$$\frac{\pi\sigma^2 a}{E} > 2\gamma_e \,, \qquad (1.7)$$

which can be rearranged to

$$\sigma\sqrt{a} > \sqrt{\frac{2E\gamma_e}{\pi}} \,. \qquad (1.8)$$

Equation (1.8) indicates that crack extension in ideally brittle materials is governed by the product of the remotely applied stress and the square root of the crack length and by material properties. Because E and γ_e are material properties the right-hand side of equation (1.8) is equal to a constant value characteristic of a given ideally brittle material. Consequently, equation (1.8) indicates that crack extension in such materials occurs when the product $\sigma\sqrt{a}$ attains a certain critical value.

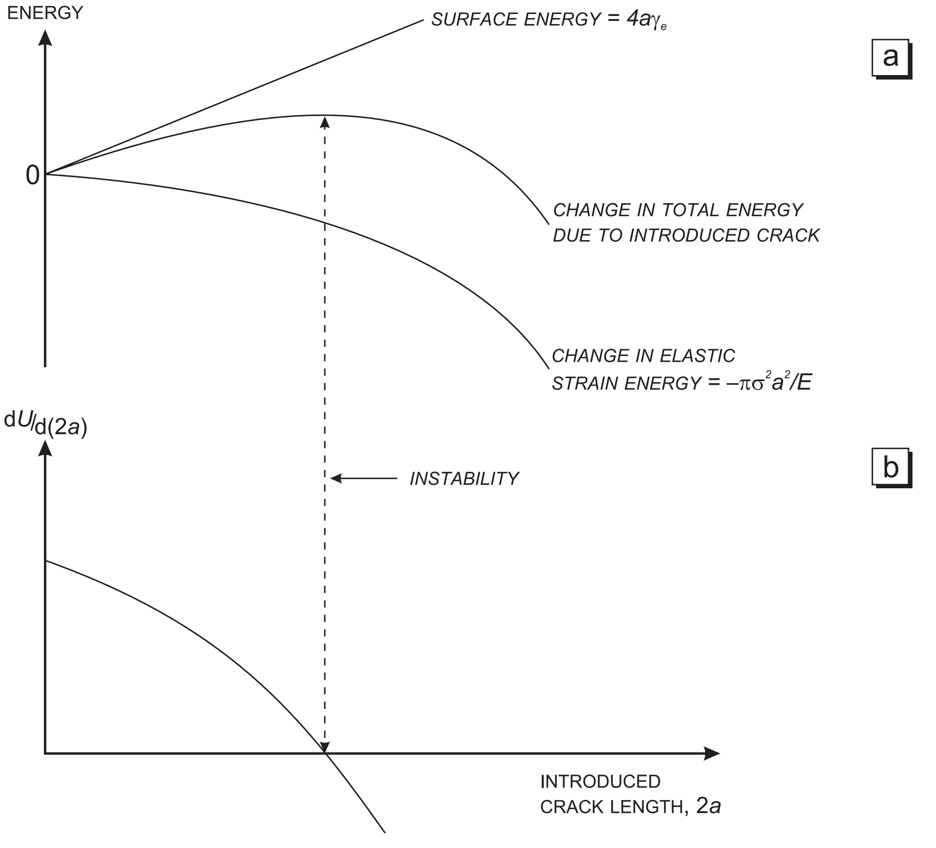

Figure 1.6. Energy balance for a small crack in a large plate loaded under fixed grip conditions.

1.5 Irwin's Modification to the Griffith Theory

Irwin designated the left-hand side of equation (1.7) as the *energy release rate*, G, representing the energy per unit new crack area that is available for infinitesimal crack extension.[2] The right-hand side of equation (1.7) represents the surface energy increase per unit new crack area that would occur owing to infinitesimal crack extension and is designated the *crack resistance*, R. It follows that G must be larger than R before crack growth occurs. If R is a constant, this means that G must exceed a critical value $G_c = R$ = constant. Thus fracture occurs when

$$G = \frac{\pi\sigma^2 a}{E} > G_c = R = 2\gamma_e \,. \tag{1.9}$$

The critical value G_c can be determined by measuring the critical stress σ_c required to fracture a plate with a crack of size $2a$ or by measuring the critical crack size $2a_c$ needed

[2] The crack area is defined as the projected area, normal to the crack plane, of the newly formed surfaces.

to fracture a plate loaded by a stress σ.

In 1948 Irwin suggested that the Griffith theory for ideally brittle materials could be modified and applied to both brittle materials and metals that exhibit plastic deformation. A similar modification was proposed by Orowan. The modification recognised that a material's resistance to crack extension is determined by the sum of the surface energy γ_e and the plastic strain work γ_p (both per unit crack surface area) that accompany crack extension. Consequently, in this case the crack resistance is

$$R = 2(\gamma_e + \gamma_p) \ . \tag{1.10}$$

For relatively ductile materials $\gamma_p >> \gamma_e$, *i.e.* R is mainly plastic energy and the surface energy can be neglected.

Although Irwin's modification includes a plastic energy term, the energy balance approach to crack extension is still limited to defining the conditions required for instability of an ideally sharp crack. Also, the energy balance approach presents insuperable problems for many practical situations, especially slow stable crack growth, as for example in fatigue and stress corrosion cracking.

The energy balance concept will be treated in more detail in chapter 4.

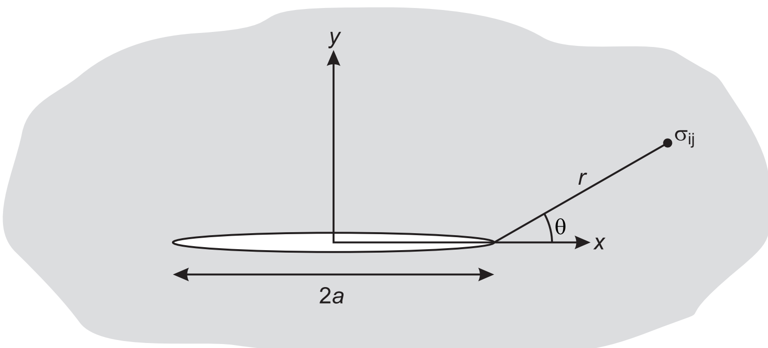

Figure 1.7. Stresses at a point ahead of a crack tip.

1.6 The Stress Intensity Approach

Owing to the practical difficulties of the energy approach a major advance was made by Irwin in the 1950s when he developed the stress intensity approach. First, from linear elastic theory Irwin showed that the stresses in the vicinity of a crack tip take the form

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \ldots \ , \tag{1.11}$$

where r, θ are the cylindrical polar co-ordinates of a point with respect to the crack tip, figure 1.7.

K is a quantity which gives the magnitude of the elastic stress field. It is called the *stress intensity factor*.[3] Dimensional analysis shows that K must be linearly related to stress and directly related to the square root of a characteristic length. Equation (1.8) from Griffith's analysis indicates that this characteristic length is the crack length, and it turns out that the general form of the stress intensity factor is given by

$$K = \sigma\sqrt{\pi a} \cdot f(a/W) \,, \tag{1.12}$$

where $f(a/W)$ is a dimensionless parameter that depends on the geometries of the specimen and crack, and σ is the (remotely) applied stress. For an infinite plate with a central crack with length $2a$, $f(a/W) = 1$ and thus $K = \sigma\sqrt{\pi a}$. For this case we also have $G = \pi\sigma^2 a/E$, see equation (1.9). Combining the two formulae for K and G yields the relation:

$$G = \frac{K^2}{E} \,, \tag{1.13}$$

which Irwin showed to be valid for any geometry.

Since $K = \sigma\sqrt{\pi a}$ for a central crack in an infinite plate, it follows from the result of Griffith's energy balance approach, equation (1.8), that crack extension will occur when K reaches a certain critical value. This value, K_c, is equal to $\sqrt{2E\gamma_e}$ or, after applying Irwin's modification, $\sqrt{2E(\gamma_e + \gamma_p)}$. The criterion for crack extension in terms of K is

$$K = \sigma\sqrt{\pi a} > K_c \,. \tag{1.14}$$

The parameter governing fracture may therefore be stated as either a critical energy release rate, G_c, or a critical stress intensity, K_c. For tensile loading the relationships between G_c and K_c are

$$G_c = \frac{K_c^2}{E} \,. \tag{1.15}$$

The value of the critical stress intensity K_c can be determined experimentally by measuring the fracture stress for a large plate that contains a through-thickness crack of known length. This value can also be measured by using other specimen geometries, or else can be used to predict critical combinations of stress and crack length in these other geometries. This is what makes the stress intensity approach to fracture so powerful, since values of K for different specimen geometries can be determined from conventional elastic stress analyses: there are several handbooks giving relationships between the stress intensity factor and many types of cracked bodies with different crack sizes, orientations and shapes, and loading conditions. Furthermore, the stress intensity factor, K, is applicable to stable crack extension and does to some extent characterize processes of subcritical cracking like fatigue and stress corrosion, as will be mentioned in section 1.10 of this chapter and in greater detail in chapters 9 and 10.

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[3] The stress intensity factor is essentially different from the well-known stress concentration factor. The latter is a dimensionless ratio that describes the increase in stress level relative to the nominal stress.

It is the use of the stress intensity factor as the characterizing parameter for crack extension that is the fundamental principle of Linear Elastic Fracture Mechanics (LEFM). The theory of Linear Elastic Fracture Mechanics is well developed and will be discussed in chapter 2.

1.7 Crack Tip Plasticity

The elastic stress distribution in the vicinity of a crack tip, equation (1.11), shows that as r tends to zero the stresses become infinite, *i.e.* there is a stress singularity at the crack tip. Since structural materials deform plastically above the yield stress, there will in reality be a plastic zone surrounding the crack tip. Thus the elastic solution is not unconditionally applicable.

Irwin considered a circular plastic zone to exist at the crack tip under tensile loading. As will be discussed in chapter 3, he showed that such a circular plastic zone has a diameter $2r_y$, figure 1.8, with

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2 , \tag{1.16}$$

where σ_{ys} is the yield stress.

Irwin argued that the occurrence of plasticity makes the crack behave as if it were longer than its physical size — the displacements are larger and the stiffness is lower than in the elastic case. He showed that the crack may be viewed as having a notional tip at a distance r_y ahead of the real tip, *i.e.* in the centre of the circular plastic zone (see figure 1.8). Beyond the plastic zone the elastic stress distribution is described by the K corresponding to the notional crack size. As shown in figure 1.8, this elastic stress distribution takes over from the yield stress at a distance $2r_y$ from the actual crack tip.

Since the same K always gives the same plastic zone size for materials with the same

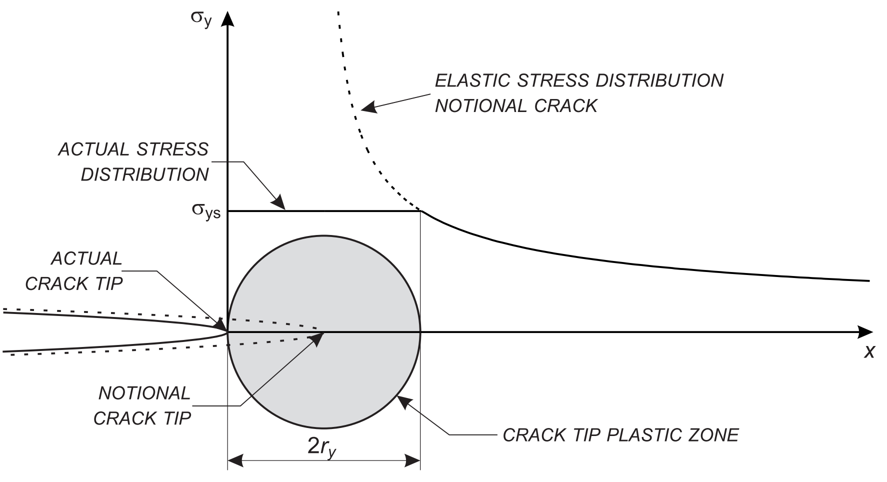

Figure 1.8. The crack tip plastic zone according to Irwin.

yield stress, equation (1.16), the stresses and strains both within and outside the plastic zone will be determined by K and the stress intensity approach can still be used. In short, the effect of crack tip plasticity corresponds to an apparent increase of the elastic crack length by an increment equal to r_y.

A plastic zone at the tip of a through-thickness crack will inevitably tend to contract in the thickness direction along the crack front. If the plate thickness is of the order of the plastic zone size or smaller, this contraction can occur freely and a *plane stress* state will prevail. On the other hand, if the plate thickness is much larger than the plastic zone size, contraction is constrained by the elastic material surrounding the plastic zone. The strain in the thickness direction will then be small, meaning that a *plane strain* state is present.[4]

The occurrence at the crack tip of either a plane stress or plane strain state has a large effect on the plastic behaviour of the material. In plane strain the plastic deformation occurs only when the stresses amply exceed the yield stress. Actually, equation (1.16) is valid for a plane stress state only. For plane strain

$$r_y = \frac{1}{2\pi}\left(\frac{K}{C\sigma_{ys}}\right)^2 , \qquad (1.17)$$

where C is usually estimated to be about 1.7. Thus in plane strain the plastic zone size is considerably smaller.

1.8 Fracture Toughness

From sections 1.6 and 1.7 it follows that under conditions of limited crack tip plasticity the parameter governing tensile fracture can be stated as a critical stress intensity, K_c. The value of K_c at a particular temperature depends on the amount of thickness constraint and thus on specimen thickness. It is customary to write the limiting value of K_c for maximum constraint (plane strain) as K_{Ic}.[5]

K_{Ic} can be considered a material property characterizing the crack resistance, and is therefore called the plane strain fracture toughness. Thus the same value of K_{Ic} should be found by testing specimens of the same material with different geometries and with critical combinations of crack size and shape and fracture stress. Within certain limits this is indeed the case, and so a knowledge of K_{Ic} obtained under standard conditions can be used to predict failure for different combinations of stress and crack size and for different geometries.

K_c can also be determined under standard conditions, and the value thus found may also be used to predict failure, but only for situations with the same material thickness and constraint.

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4 In all formulae up to this point a plane stress state was implicitly assumed.

5 The subscript I refers to the loading mode where the crack flanks are pulled straight apart (see section 2.1). In fracture mechanics it is customary to include this subscript in expressions that contain the stress intensity factor as a variable, i.e. K_I. However, in this introductory chapter this is not yet done.

As an introductory numerical example of the design application of LEFM, consider the equation for a through-thickness crack in a wide plate, *i.e.*

$$K = \sigma\sqrt{\pi a}\ . \tag{1.18}$$

Assume that the test results show that for a particular steel the K_c is 66 MPa$\sqrt{\text{m}}$ for the plate thickness and temperature in service. Using equation (1.18) a residual strength curve for this steel can be constructed relating K_c and nominal stress and crack size. This is shown in figure 1.9. Also assume that the design stress is 138 MPa. It follows from equation (1.18) and figure 1.9 that the tolerable crack size would be about 145 mm. For a design stress of 310 MPa the same material could tolerate a crack size of only about 28 mm. Note from figure 1.9 that if a steel with a higher fracture toughness is used, for example one with a K_c of 132 MPa$\sqrt{\text{m}}$, the permissible design stress for a given crack size is significantly increased. *Thus a material with a higher fracture toughness permits a longer crack at a given stress or a higher stress at a given crack length.*

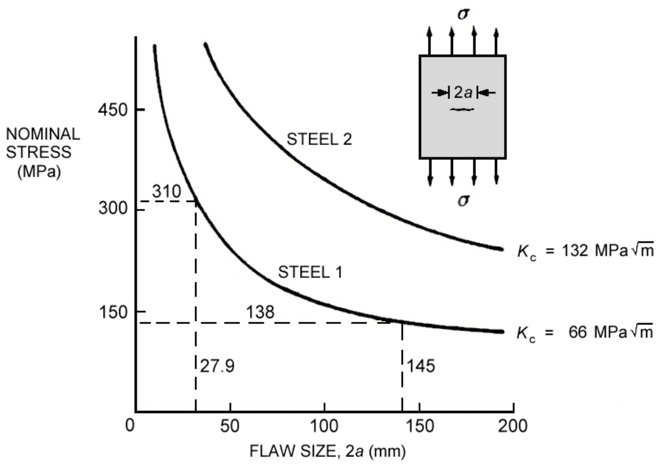

Figure 1.9. Residual strength curves for two steels.

1.9 Elastic-Plastic Fracture Mechanics

Linear Elastic Fracture Mechanics can deal with only limited crack tip plasticity, *i.e.* the plastic zone must remain small compared to the crack size and the cracked body as a whole must still behave in an approximately elastic manner. If this is not the case then the problem has to be treated elasto-plastically. Due to its complexity the concepts of Elastic-Plastic Fracture Mechanics (EPFM) are not so well developed as LEFM theory, a fact that is reflected in the approximate nature of the eventual solutions.

In 1961 Wells introduced the crack opening displacement (COD) approach. This approach focuses on the strains in the crack tip region instead of the stresses, unlike the stress intensity approach. In the presence of plasticity a crack tip will blunt when it is loaded in tension. Wells proposed to use the crack flank displacement at the tip of a blunting crack, the so-called crack tip opening displacement (CTOD) as a fracture parameter (see figure 1.10).

Even for tougher materials exhibiting considerable plasticity critical CTOD values could be defined corresponding to the onset of fracture. Such critical CTOD values

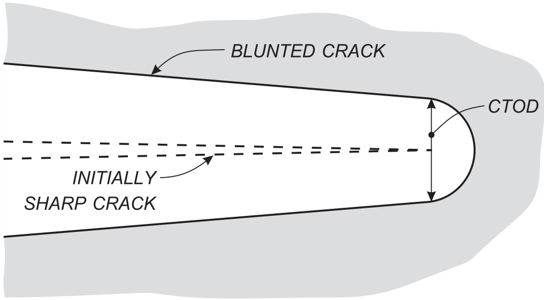

Figure 1.10. Crack tip opening displacement.

could then be used to qualify the materials concerned for a given application. However, initially it proved difficult to determine the required CTOD for a given load and geometry or alternatively to calculate critical crack lengths or loads for a given material.

In 1968 Rice considered the potential energy changes involved in crack growth in non-linear elastic material. Such non-linear elastic behaviour is a realistic approximation for plastic behaviour provided no unloading occurs in any part of the material. Rice derived a fracture parameter called J, a contour integral that can be evaluated along any arbitrary path enclosing the crack tip, as illustrated in figure 1.11. He showed J to be equal to the energy release rate for a crack in non-linear elastic material, analogous to G for linear elastic material.

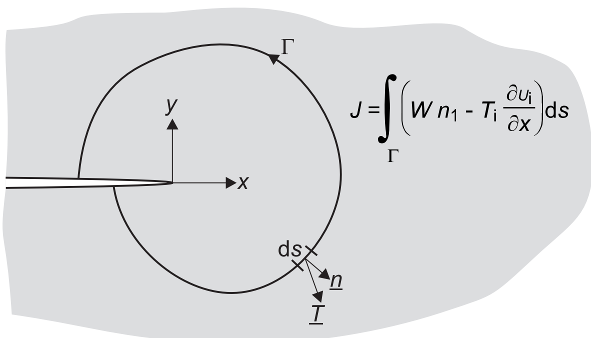

Figure 1.11. J contour integral along arbitrary path Γ enclosing a crack tip in non-linear elastic material. W is strain energy density along Γ, $\underline{n}$ is outward-directed unit vector normal to Γ, $\underline{T}$ is traction acting on Γ and $\underline{u}$ is the displacement along Γ.

For simple geometries and load cases the J integral can be evaluated analytically. However, in practice finite element calculations are often required. In spite of this J has found widespread application as a parameter to predict the onset of crack growth in elastic-plastic problems. Later it was found that J could also be used to describe a limited amount of stable crack growth.

In chapter 6 the background to the J and COD approaches are discussed, while chapter 7 deals with the procedures to measure critical values of these parameters in

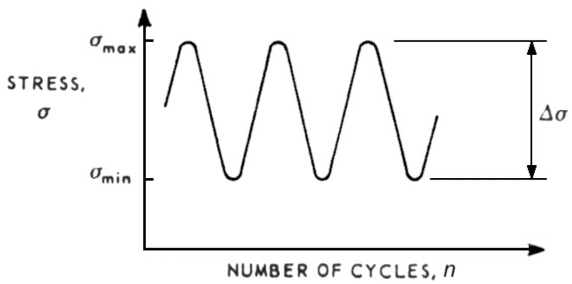

Figure 1.12. Stress-cycle parameters in constant amplitude fatigue.

actual materials. In chapter 8 some specific aspects of EPFM are discussed.

1.10 Subcritical Crack Growth

In section 1.7 it was mentioned that the stress intensity factor can still be used when crack tip plasticity is limited. This latter condition holds for some important kinds of subcritical crack growth, where most of the crack extension usually takes place at stress intensities well below K_c and K_{Ic}. In particular the stress intensity approach can provide correlations of data for fatigue crack growth and stress corrosion cracking.

Fatigue

Consider a through-thickness crack in a wide plate subjected to remote stressing that varies cyclically between constant minimum and maximum values, *i.e.* a fatigue loading consisting of constant amplitude stress cycles as in figure 1.12. The stress range $\Delta\sigma = \sigma_{max} - \sigma_{min}$, and from equation (1.18) a stress intensity range may be defined:

$$\Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi a} \ . \tag{1.19}$$

The fatigue crack growth rate is defined as the crack extension, Δa, during a small number of cycles, Δn, *i.e.* the growth rate is $\Delta a/\Delta n$, which in the limit can be written as the differential $\mathrm{d}a/\mathrm{d}n$. It has been found experimentally that provided the stress ratio, $R = \sigma_{min}/\sigma_{max}$, is the same then ΔK correlates fatigue crack growth rates in specimens with different stress ranges and crack lengths and also correlates crack growth rates in specimens of different geometry, *i.e.*

$$\frac{\mathrm{d}a}{\mathrm{d}n} = f(\Delta K, R) \ . \tag{1.20}$$

This correlation is shown schematically in figure 1.13. Note that it is customary to plot $\mathrm{d}a/\mathrm{d}n - \Delta K$ data on a double logarithmic scale. The data obtained with a high stress range, $\Delta\sigma_h$, correspond to a lower critical crack length and commence at relatively high values of $\mathrm{d}a/\mathrm{d}n$ and ΔK. The data for a low stress range, $\Delta\sigma_l$, commence at lower values of $\mathrm{d}a/\mathrm{d}n$ and ΔK, but reach the same high values as in the high stress range case. The data frequently show a sigmoidal trend, and this will be discussed in chapter 9 together

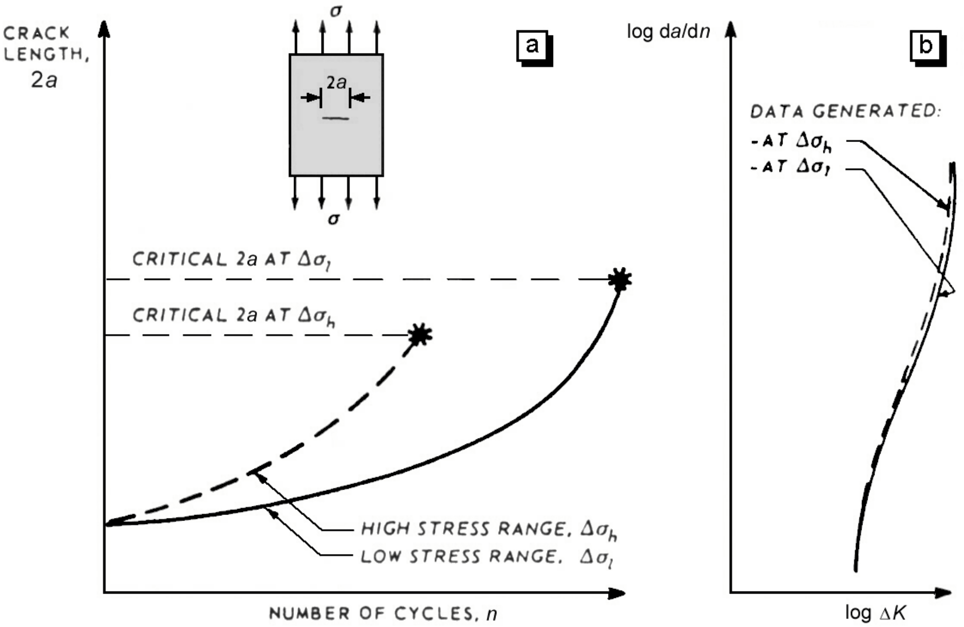

Figure 1.13. Correlation of fatigue crack propagation data by ΔK when the stress ratio, R, is the same.

with additional aspects of fatigue crack growth.

Stress Corrosion

It has also been found that stress corrosion cracking data may be correlated by the stress intensity approach. Figure 1.14 gives a generalised representation of the stress corrosion crack growth rate, da/dt, as a function of K, where t is time.

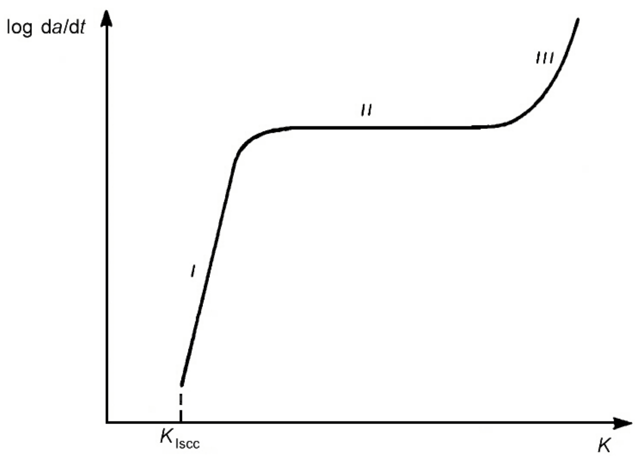

Figure 1.14. Stress corrosion crack growth rate as a function of K.

The crack growth curve consists of three regions. In regions I and III the crack velocity depends strongly on K_I, but in region II the velocity is virtually independent of K_I. Regions I and II are most characteristic. Region III is often not observed owing to a fairly abrupt transition from region II to unstable fast fracture. In region I there is a so-called threshold stress intensity, designated $K_{I_{scc}}$, below which cracks do not propagate under sustained load for a given combination of material, temperature and environment. This threshold stress intensity is an important parameter that can be determined by time-to-failure tests in which pre-cracked specimens are loaded at various (constant) stress intensity levels, thereby failing at different times as shown schematically in figure 1.15.

The subject of stress corrosion cracking, under the more general heading of sustained load fracture, will be examined further in chapter 10.

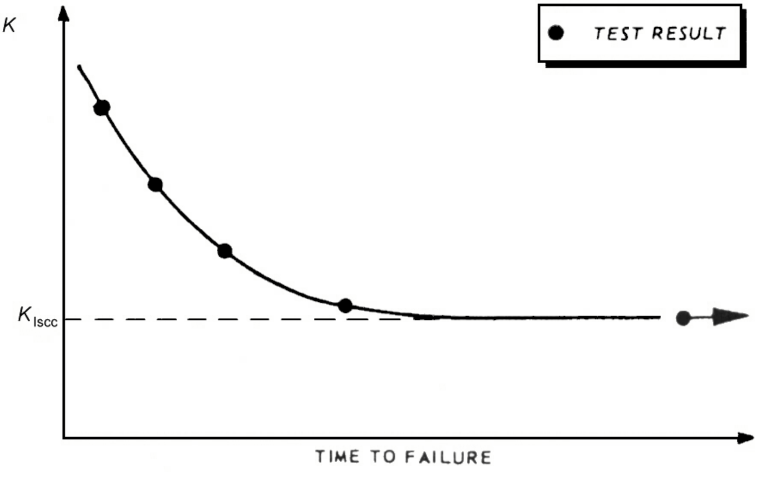

Figure 1.15. Schematic time-to-failure curve with $K_{I_{scc}}$.

1.11 Influence of Material Behaviour

So far, this overview of the use of fracture mechanics to characterize crack extension has not taken account of actual material behaviour, the influence of which may be considerable. For example, the fracture toughness of a material is much less when crack extension occurs by cleavage instead of ductile fracture. Cleavage is an intrinsically brittle mode of fracture involving separation of atomic bonds along well-defined crystallographic planes.

Other examples of material behaviour that affect fracture properties are:

1. Cracking of second phase particles in the metallic matrix and formation of microvoids at particle/matrix interfaces.
2. Anisotropic deformation and fracture. This may be intrinsic (crystallographic) as in the case of cleavage, or may result from material processing (texture).
3. Choice of fracture path, *i.e.* whether transgranular or intergranular, or a mixture of both.